Persistence in North American Palmer Drought Severity Index Data Reconstructed from Tree Ring History

G. Padmanabhan[1]*, S. L. Shrestha[2], and R. G. Kavasseri[3]
* Corresponding author; 1 & 2: Dept. of Civil Engineering, 3: Dept. of Electrical Engineering
North Dakota State University
Fargo, North Dakota, U.S.A.

Introduction

Societal vulnerability to droughts with devastating consequences has come under scrutiny time and again in the history of mankind. Recent droughts in the North American Plains are no exceptions. The resulting acute water shortages and lower or no crop yields have significant impact on the economy. The magnitude of impact on society and economy depends on the severity, duration and the spatial extent of droughts (Dracup et al., 1980). In general, droughts are associated with water deficit. However, drought can be defined from several perspectives (Wilhite and Glantz, 1987; Changnon, 1987). For example, a large deficit of precipitation from normal may be referred to as meteorological drought, of stream flows and groundwater levels as hydrological drought, and of soil moisture availability as agricultural drought (Yevjevich, 1967). If precipitation is the carrier of drought signal, stream flow, groundwater level, soil moisture availability, etc., are drought indicators (Klemes, 1987). One of the popular drought indicators used for drought management and mitigation is Palmer Drought Severity Index (PDSI) and has been analyzed by many investigators for temporal and spatial characteristics (Palmer, 1965; Karl, 1986; Karl et al., 1987; Wilhite and Glantz, 1987, Guttman, 1991 and 1998; Guttman et al., 1992; Heddinghaus and Sabol, 1991). Climate is the driving mechanism of droughts. Therefore, a better understanding of the past spatial and temporal variability of climate and drought indicators is essential to develop better perspectives of long-term variations and correlative structure of the drought characteristics. Drought time series from different parts of the world have been analyzed extensively in the last three decades using traditional time series analysis techniques (Chin and Yevjevich, 1974, Yevjevich, 1977; Padmanabhan and Rao, 1979 and 1988; Rao and Padmanabhan, 1984). Typically, these investigations focused on identifying dominant frequency components in the climatological and hydrological time series for improving modeling and simulation of hydrological variables with exogenous input (Padmanabhan, 1990 and 1991). Hydrologic variables over longer time scales could be governed by differing processes mechanisms. Therefore, trends and the correlations need to be separated to analyze long term correlations. Though classical tools such as autocorrelation function and spectral analysis can provide preliminary indications for the presence of long range correlation, it may be inappropriate to use them to determine temporal scaling properties, particularly in the presence of non-stationarities (Kavasseri and Nagarajan, 2005). In recent years several advancements have been made, particularly in the investigation of long-term persistence or correlations and temporal scaling properties of time series data from diverse disciplines (Ivanova and Ausloos, 1999; Peng et al., 1995; Hausdorff et al., 1995; Vandewalle and Ausloos, 1997; Kantelhardt et al., 2001; Kavasseri and Nagarajan, 2005). The significance of long-range persistence of droughts in drought hazard assessment and management has been recognized and addressed by many researchers (Bras and Rodriguez-Iturbe, 1985; Pellettier and Turcotte, 1997). However, the recent developments in time series analysis techniques have not been taken full advantage of in further investigating the long-term persistence and temporal scaling characteristics of drought time series.

Hydrologists and water resource managers have always been interested in quantifying impacts of climate variability on water resources. In a sense, hydrologists were among the first scientists to define the first order impacts of climate so as to design water structures, operate water facilities, and to assess weather events affecting water supplies (Changnon, 1987). However, most hydrologic studies of extreme events such as floods and droughts have assumed stationarity of climate over time. Several researchers have pointed out that non-random climate shifts do occur and we need to include them in the planning and design of water resources systems (Chin and Yevjevich, 1974; Yevjevich, 1977). Nemec and Schaake (1982) found that moderate fluctuations in climate may produce major hydrologic changes. Meko and Stockton (1984) found that stationarity assumption clearly did not apply in the western and Changnon (1985) in mid-western United States. More recently, it has been emphasized that stationarity assumptions should no longer serve as a central, default assumption in water resources management because of the hydroclimatic change apparently now under way (Milly et al., 2008).

In this paper we analyze some Palmer Drought Severity Index (PDSI) time series reconstructed from tree ring data for their temporal scaling properties and long-term persistence using Detrended Fluctuation Analysis (DFA) (Peng et al., 1994).

Data

The data for this study was obtained from 'North American Drought Atlas' which provides 286 annual tree-ring drought reconstructions on a 2.5 degree by 2.5 degree grid network of summer Palmers Drought Severity Index (PDSI) (Cook and Krusic, 2004; US Department of Commerce, 2007). The data length of the reconstructed PDSI varies from about 300 to 2000 years at some locations. Locations with longest reconstructed data are concentrated in an area bounded between 122.5 W to 102.0 W and from 42.5 N to 30.0 N, mainly in California, Nevada, Utah, Colorado, and New Mexico (Fig. 1)

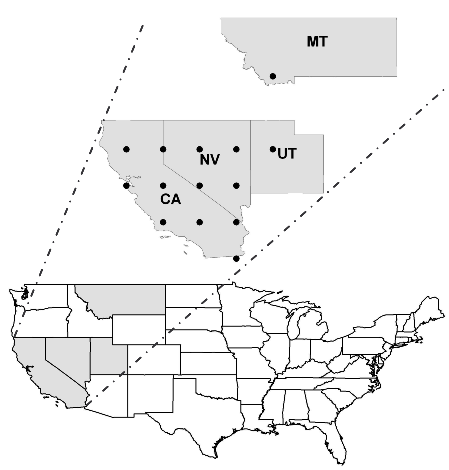

Fig 1: US region selected for analysis. Inset shows selected stations with longest PDSI reconstructed available data.

Table 1: Selected stations.

Station ID	State	Latitude	Longitude	No of years of reconstructed PDSI data used in analysis
035	CA	N 40.0	W 122.5	2000 (1 to 2000)
036	CA	N 37.5	W 122.5	2000 (1 to 2000)
046	CA	N 40.0	W 120.0	2000 (1 to 2000)
047	CA	N 37.5	W 120.0	2000 (1 to 2000)
048	CA	N 35.0	W 120.0	2000 (1 to 2000)
058	NV	N 40.0	W 117.5	2000 (1 to 2000)
059	NV	N 37.5	W 117.5	2000 (1 to 2000)
060	CA	N 35.0	W 117.5	2000 (1 to 2000)
071	NV	N 40.0	W 115.0	2000 (1 to 2000)
072	NV	N 37.5	W 115.0	2000 (1 to 2000)
073	CA	N 35.0	W 115.0	2000 (1 to 2000)
074	CA	N 32.5	W 115.0	2000 (1 to 2000)
084	MT	N 45.0	W 112.5	2004 (1 to 2000)
086	UT	N 40.0	W 112.5	2004 (1 to 2000)

Methods Used

Power spectral techniques have been traditionally used in the past to analyze hydrologic and climatologic time series data for their frequency content and to detect possible long-range correlations of the power-law form (Feder, 1988).

Hurst exponent (H) is a parameter indicative of the nature of correlative properties of time series data. Hurst (1951) and Hurst et al. (1965) studied the long-term correlations using Rescaled Range technique and showed that many hydrologic and climatologic time series exhibit a power-law relationship with an average exponent of 0.73; a value greater than 0.5 indicates the presence of long term persistence. Other methods to estimate the Hurst exponent include variance method, Whittle estimator, and wavelet based methods. Hurst estimators are sensitive to trends in the data and therefore, may give spurious results. Several interpretations have been put forth by various investigators. For example, large-scale variability of a time series can explain Hurst phenomenon (Koutsoyiannis, 2000 and 2002). It can be due to a mixture of temporal scales (Mesa and Poveda, 1993).

Recently the Detrended Fluctuation Analysis (DFA) and its extensions have been proposed as an alternative to infer possible long range correlations and temporal scaling properties in data sets obtained from diverse disciplines (Ivanova and Ausloos, 1999; Peng et al., 1995; Hausdorff et al., 1995; Vandewalle and Ausloos, 1997; Kantelhardt et al., 2001; Kavasseri and Nagarajan, 2005). In this method, in order to remove the seasonal trends, the fluctuations in the data rather than the actual data are used.

Power spectral density (PSD) estimation, Hurst H, and Detrended Fluctuation Analysis were used in this study. Welch method (Welch, 1967) was used for power spectral estimation. Rescaled Range analysis was used to determine the Hurst exponent H.

Data Analysis

Time series and Power Spectrum

Figure 2 shows a temporal trace and power spectrum of the reconstructed PDSI of a representative record for a location 112.5 W and 45.0 N in the United States.

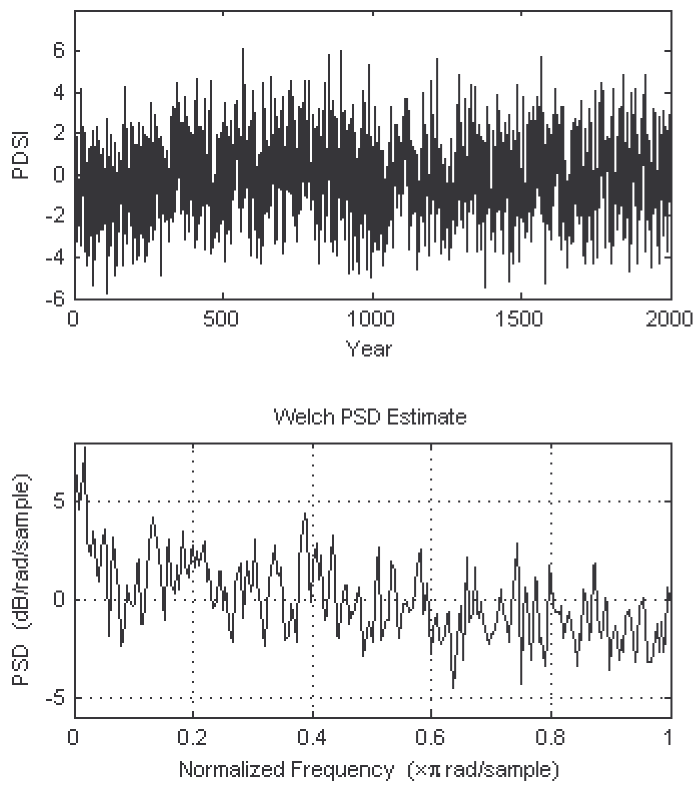

Figure 2: Temporal trace and Power spectrum of a representative record (Station 084)

Use of PDSI offers a number of advantages compared to seasonal temperature or precipitation values as it integrates both temperature and precipitation to estimate how the soil moisture availability differs from normal condition (Brewer et al., 2007). The index ranges between negative values for dry conditions and positive values for wet conditions; values less than -4 in indicate extreme drought where as greater than +4 would indicate an extreme wet spell.

Rescaled Range

Rescaled range, also called as R/S statistic, is defined as the ratio $R/S = R(k)/S(k)$ where $R(k)$ is the sequential range for sample k, and $S(k)$ is the square root of sample sequential variance $S^2(k)$. The ratio R/S varies as k^H. If we plot R/S against k in a log-log paper the slope of the linear relationship should yield an estimate of H.

Rescaled Range analysis was carried out on data from all stations to estimate their Hurst exponents. Sample size of multiple of 100 was used for the analysis for a total of 2000 data, giving 20 data points. For long-memory process, the points in the R/S plot should be scattered randomly around a straight line with a slope H > 0.5. To be more precise, the points should ultimately (for large values of k) be scattered randomly around a straight line with a slope H > 0.5, for sufficiently large lags. In present analysis, with a total data points in the neighborhood of 2000, "large" value of k was arbitrarily taken as greater than 1000.

Table 2: R/S range analysis values

Station ID	State	Latitude	Longitude	Overall slope (H)	Slope for k >1000
035	CA	N 40.0	W 122.5	1.1003	0.6819
036	CA	N 37.5	W 122.5	1.0674	0.6658
046	CA	N 40.0	W 120.0	0.9961	0.6651
047	CA	N 37.5	W 120.0	1.0701	0.9247
048	CA	N 35.0	W 120.0	0.8340	0.3764
058	NV	N 40.0	W 117.5	0.8927	0.4153
059	NV	N 37.5	W 117.5	0.8120	0.5953
060	CA	N 35.0	W 117.5	0.7000	1.0303
071	NV	N 40.0	W 115.0	0.5793	0.0131
072	NV	N 37.5	W 115.0	0.5633	0.5328
073	CA	N 35.0	W 115.0	0.5384	1.4177
074	CA	N 32.5	W 115.0	0.7988	1.7390
084	MT	N 45.0	W 112.5	0.9010	-0.0744
086	UT	N 40.0	W 112.5	0.5099	0.7914

Figure 3(a) presents the scatter of the points around a straight line with a slope greater than 0.5 for station 035 while Figure 3(b) shows the scatter of the points for $k \rangle 1000$.

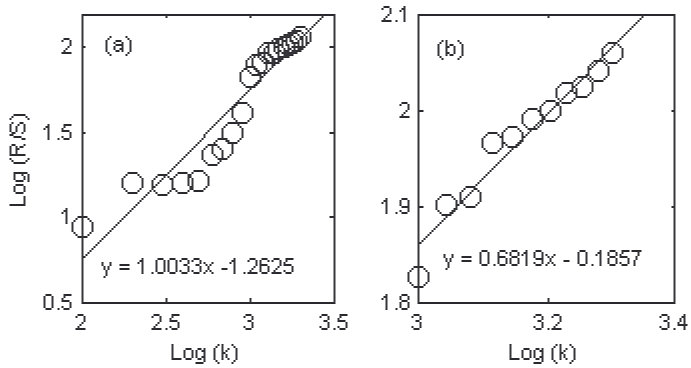

Figure 3: Estimation of Hurst's coefficient for a representative station (Station 035): (a) Overall scatter (b) scatter of points with $k \geq 1000$

Detrended Fluctuation Analysis

In order to remove the seasonal trends, the fluctuations in the PDSI data rather than the actual PDSI value was used in the DFA analysis.

$$\Delta PDSI_i = PDSI_i - \langle PDSI_i \rangle$$

In this equation, $\langle PDSI_i \rangle$ is the mean value of PDSI for the whole period. A running sum of the PDSI fluctuation is calculated as,

$$y(m) = \sum_{i=1}^{n} \Delta PDSI_i$$

where m = 1, …, n. The time series of the y (m) is next divided into non-overlapping intervals with equal lengths n. In each interval, y (m) is fitted to a straight line, x (m) = km + d, for each segment and the detrended square variability $F^2(n)$ is calculated as

$$F^2(n) = \left\langle \frac{1}{n} \sum_{m=kn+1}^{(k+1)n} \left(y(m) - x(m) \right)^2 \right\rangle$$

with

$$k = 0,1,2,\ldots,\left(\frac{N}{n} - 1 \right).$$

If the PDSI fluctuation were uncorrelated, indicating a white noise, one expects $F(n) \approx n^{\alpha}$ where $\alpha = \frac{1}{2}$. If $\alpha \rangle \frac{1}{2}$, one can expect long-range power law correlations in the data for the range of values considered.

Reconstructed PDSI time series from 14 stations were analyzed using the DFA algorithm (Peng et al., 1994). A fourth order polynomial was used for regression in the DFA algorithm. The result of the analysis is presented in Table 3.

Table 3. Scaling factor for selected PDSI data in the mid-western USA.

Station ID	State	Latitude	Longitude	Scaling factor (α)
035	CA	N 40.0	W 122.5	0.4662
036	CA	N 37.5	W 122.5	0.4926
046	CA	N 40.0	W 120.0	0.4893
047	CA	N 37.5	W 120.0	0.5157
048	CA	N 35.0	W 120.0	0.5428
058	NV	N 40.0	W 117.5	0.5646
059	NV	N 37.5	W 117.5	0.5656
060	CA	N 35.0	W 117.5	0.5660
071	NV	N 40.0	W 115.0	0.6505
072	NV	N 37.5	W 115.0	0.6399
073	CA	N 35.0	W 115.0	0.6722
074	CA	N 32.5	W 115.0	0.6192
084	MT	N 45.0	W 112.5	0.6547
086	UT	N 40.0	W 112.5	0.6731

Table 3 shows that out of a total of 14 data sets, 10 data sets (71%) clearly show the scaling index greater than 0.5, indicating long term memory. However, 4 data sets show the scaling index less than 0.5. A plot of $\log_{10}F(n)$ vs. $\log_{10}(n)$ is shown for representative stations 036, 047, and 084 in Figure 4. The slope of these plots gives the value of alpha.

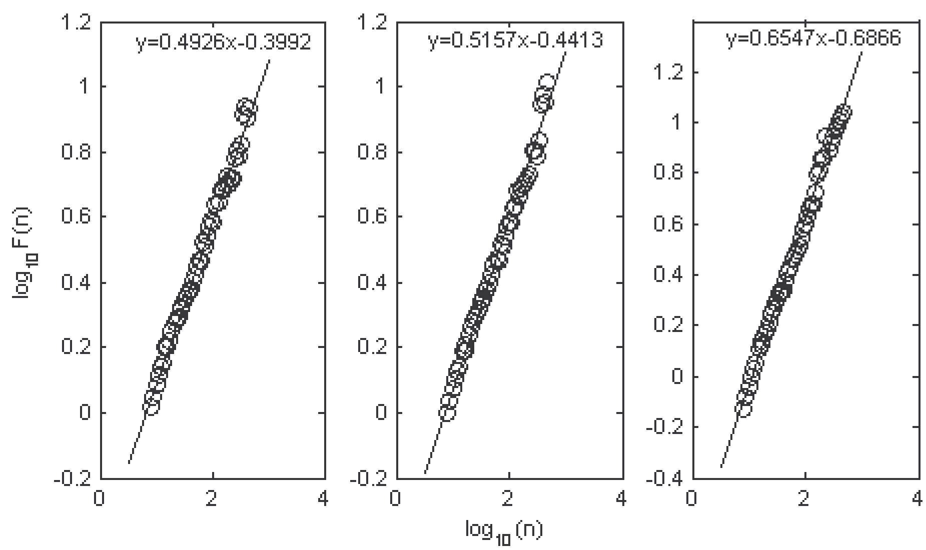

Figure 4. A log-log plot of the DFA analysis for three representative stations:
(a) Station 036 (b) Station 047 (c) Station 084.

Discussion

Examination of the power spectrum of the PDSI in Figure 2 indicates a power-law decay which is suggestive of a possible long-range correlation. Long range correlations generally indicate that temporally well separated samples of time series are correlated with each other and indicative of a self-similar behavior.

In the Rescaled Range analysis H is estimated by the slope of the straight line showing the "ultimate" behavior of the data. An inherent difficulty in this method is to decide from which k value on the "ultimate behavior" starts (Baren, 1994). In the values obtained in the present analysis, overall slope values are greater than 0.5 in all stations as shown in Table 2. Analyzing with k>1000, majority of the stations shows H value greater than 0.5, with 10 out of 14 stations. However, for two stations the value is well below 0.5. Thus it gives uncertainty in the estimation of H value.

The techniques used in the present study, viz. R/S Analysis and DFA analysis are used quite extensively, however, these techniques can extract only a single scaling exponent from a time series and therefore appropriate only for the analysis of monofractal time series which have uniform scaling properties throughout the signal which can be characterized by a single exponent. It is possible that some processes may be governed by more than one scaling exponent, in which case a single scaling exponent would be unable to capture the complex dynamics inherent in the data. Multifractal Detrended Fluctuation Analysis (MFDFA), though far more complex than monofractal ones, may be used to investigate the scaling properties to rule out multiple scaling effects (Barabasi and Vicsek, 1991; Kantelhardt et al., 2000 and 2002; Kavasseri and Nagarajan, 2005). The data sets used in the present study were limited to 2000 data points. It has been pointed out earlier that short data lengths can induce finite size effects in to the results obtained from time series analysis tools such as DFA. Thus, further analysis is required on longer data sets to rigorously corroborate the scaling results obtained in this study.

Conclusion

DFA technique has considerable potential for investigating long-term memories in hydrological processes. Though it has been used in some hydrological contexts, to our knowledge, it has not been used to analyze long-term memory in drought data. Our analysis shows that the reconstructed PDSI data for the 14 selected stations, where long spans of data were available, had the self-similarity parameter, or the scaling exponent values between 0.5 and 1, thus showing the possibility of long term memory in the drought occurrence. Generalization is possible only after analysis of additional data from other locations. Nonetheless, the DFA method holds strong promise for use in long-term memory investigations of hydrological processes

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