HYDROLOGIC FORECASTING OF FAST FLOOD EVENTS IN SMALL CATCHMENTS WITH A 2D-SWE MODEL. NUMERICAL MODEL AND EXPERIMENTAL VALIDATION.

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This paper presents the application of an unstructured finite-volume two-dimensional shallow water model in order to compute the precipitation runoff in small catchments. The depth averaged mass and momentum conservation equations are solved, considering the effects of bed friction, bed slope, precipitation and infiltration. Hence, model has a conceptual and mathematical ground much stronger than classic hydrologic models. The numerical problems which arise from the small water depths, the presence of wet-dry fronts and the large bed friction forces, are successfully solved in the present model. The only parameters of the model which need calibration are the bed friction coefficient and the infiltration properties of the soil. The effects of small scale microtopography which is not resolved by the model, must be included via the bed friction coefficient, in the same way as the effects of ripples and dunes in rivers are included in the bed friction stress. The sensitivity of the model to bed friction has been studied. In order to avoid the uncertainty in the parameter determination of a real catchment, experimental validation of the model is presented in simple 1D and 2D geometries. The model presented might be used in the forecasting of flood regions in catchments due to storm events, as well as in the design of hydraulic structures to mitigate and control flood risks.

Keywords: rainfall-runoff forecasting, hydrological model, fast flood, overland flow, rainfall, 2D shallow water equations, watershed modelling

1. INTRODUCTION

Shallow water models based on the depth averaged shallow water equations (2D-SWE) are extensively used to compute the flow field and flood regions in rivers and coastal regions. The input data required in such models are the topography, the bed roughness coefficient, and the water discharge flowing through the reach under study. The water discharge is usually obtained using simple hydrologic models based on empirical formulae, which relate the water discharge with the precipitation intensity, the catchment surface, mean slope and soil. Usually such models contain simplified representations of surface runoff, evapotranspiration, and channel flow. In addition, the water discharge is introduced in the hydraulic model at one specific cross section, while in reality there is a continuous spatially distributed contribution of runoff to the river discharge.

Recent advances in the numerical schemes for solving the two-dimensional shallow water equations permit modelling water flow over complex topography with extremely small water depths, including the propagation of wet-dry fronts. This fact, together with the continuous increase in computational efficiency, opens up the possibility of computing the surface runoff due to precipitation in a whole watershed. Precipitation surface runoff is actually a very shallow water flow, and therefore it should be well represented by the two-dimensional shallow water equations.

Zhang and Cundy (1989) used a finite-difference 2D shallow water model to simulate the rainfall-runoff experiments performed by Iwagaki (1955) in a three-slope laboratory flume. Runoff laboratory experiments over simple geometries were also modelled more recently by Yan and Kahawita (2000, 2007), obtaining some encouraging results. Howes et al. (2006) modelled the overland flow generated by two different storms in two very small semiarid watersheds using a 2D model based on the kinematic wave approximation. The watersheds studied by Howes et al. had a surface of approximately 2000m^2. The runoff hydrographs obtained with their numerical model show a rather good agreement with experimental results, especially regarding the peak discharge. Kivva and Zheleznyak (2005) modelled the rainfall runoff in a catchment watershed somewhat larger than the ones studied by Howes et al. (0.085Km^2), using a finite-difference 2D shallow water model. The agreement between experimental and numerical results of their model depends on the storm considered. All these experimental validations, which were obtained in very small catchment areas as well as in laboratory experiments over simple geometries, suggest that 2D models could be used in larger and more complex watersheds with a highly irregular surface roughness.

A research line in this direction has been recently started by the authors of this paper, which aims to validate the applicability of 2D shallow water models to forecast fast flood events in different kind of small catchments (up to 100 Km^2). This paper presents some preliminary results which include rainfall runoff experimental results obtained in a 2D laboratory model, numerical validation of an unstructured finite volume 2D shallow water model, and the numerical results obtained in a real catchment of approximately 5Km^2. At the present time, field work is being undertaken in order to validate the numerical results.

2. NUMERICAL MODEL

A complete description of the numerical model used in this paper (Turbillon) can be found in (Cea, 2007). This section presents just a brief description of the equations and numerical schemes solved by the numerical model.

The numerical model Turbillon is based on the depth averaged shallow water equations (2D-SWE), also known as 2D St. Venant equations. The 2D-SWE are obtained after vertical integration of the 3D Reynolds Averaged Navier-Stokes equations over the water depth. The main simplifications made in the 2D-SWE are the assumptions of an hydrostatic pressure distribution (the dynamic pressure is neglected) and an homogeneous velocity profile in the vertical direction (the dispersion terms which appear due to non-uniformities of the velocity in the vertical direction are neglected). Both hypotheses are fulfilled in very shallow flows, as it is the case in the applications presented in this paper, which justifies the use of a depth averaged shallow water model to simulate overland flow caused by rainfall.

The 2D-SWE are a system of three partial differential equations with three unknowns which are defined over a 2D spatial domain. Neglecting the variations in atmospheric pressure over the spatial domain, the wind stress, the Coriolis acceleration, as well as the viscous and turbulent horizontal stresses, the 2D-SWE can be written in conservative form as:

$$\frac{\partial h}{\partial t}+\frac{\partial q_x}{\partial x}+\frac{\partial q_y}{\partial y}=i-f$$

$$\frac{\partial q_x}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q_x^2}{h}+g\frac{h^2}{2}\right)+\frac{\partial}{\partial y}\left(\frac{q_x q_y}{h}\right)=-gh\frac{\partial z_b}{\partial x}-\frac{\tau_{b,x}}{\rho}+i\cdot v_x-f\cdot\frac{q_x}{h}$$

$$\frac{\partial q_y}{\partial t}+\frac{\partial}{\partial x}\left(\frac{q_x q_y}{h}\right)+\frac{\partial}{\partial y}\left(\frac{q_y^2}{h}+g\frac{h^2}{2}\right)=-gh\frac{\partial z_b}{\partial y}-\frac{\tau_{b,y}}{\rho}+i\cdot v_y-f\cdot\frac{q_y}{h}$$

where q_x, q_y are the two components of the unit discharge, h is the water depth, z_b is the bed elevation, $\tau_{b,x}$, $\tau_{b,y}$ are the two horizontal components of the bed friction stress, ρ is the water density, g is the gravity acceleration, i is the rainfall intensity, v_x,v_y are the velocity components of the rainfall, and f is the infiltration rate.

For the present applications, the effects of bed friction, bed slope, precipitation and infiltration are considered in the model. The fact of neglecting the turbulent horizontal stresses is justified in this case because the turbulent vertical shear caused by bed friction is much larger than the horizontal turbulent shear. In the 2D-SWE the turbulent vertical shear is introduced in the bed friction term, which in this case is computed with Manning's formula

$$\frac{\tau_b}{\rho}=g\frac{n^2U^2}{h^{4/3}}$$

The infiltration rate, expressed in m/s, is computed at each spatial point using the formulation of Green-Ampt (Chow, 1988), in which it is assumed that there is a saturation front in the soil which separates the saturated region beneath the bed surface and the non-saturated region, in which there is a suction ψ. As the infiltration increases, the saturation front descends and the width of the non-saturated region *L* gets larger. The potential infiltration rate *f* is then computed as:

$$f(\mathbf{x},t)=k_s(\mathbf{x})\left(1+\frac{\left(h(\mathbf{x},t)+\Psi(\mathbf{x})\right)\Delta\theta}{F(\mathbf{x},t)}\right)\qquad F(\mathbf{x},t)=\int_0^t f(\mathbf{x},t)\,dt\qquad L(\mathbf{x},t)=\frac{F(\mathbf{x},t)}{\Delta\theta}\qquad \Delta\theta=\phi-\theta_i$$

being k_s the saturated permeability of the soil, *h* the water depth over the bed surface, ψ the suction in the non-saturated region of the soil, $\Delta\theta$ the change in moisture content of the soil as the saturation front advances, θ_i the initial moisture content of the soil, ϕ the soil porosity, and *L* the width of the saturated region in the soil. The actual infiltration rate is equal to the potential infiltration rate except in the case that the water depth is too small and there is not enough water to infiltrate, in which case the infiltration is computed from the available water depth.

Other mass losses as evapotranspiration, interception and retention can also be considered in the model. However, in the applications presented in this paper these losses are not significant, and do not need to be considered in the computations.

The only parameters of the model which need calibration are the bed friction coefficient and the infiltration properties of the soil. The effects of small scale microtopography which is not resolved by the model, must be included via the bed friction coefficient, in the same way as the effects of ripples and dunes in rivers are included in the bed friction stress. By microtopography we mean the bed surface features with a length scale smaller than the mesh size used in the numerical discretisation.

An unstructured finite-volume solver with a first order explicit time discretisation is used to solve the mean flow equations. The convective flux is discretised with either a first order scheme, an hybrid second-order/first-order scheme (first order in the water depth and second order in the unit discharge), or a fully second order scheme, all of them upwind Godunov's schemes based on Roe's average (Toro, 2001). The hybrid scheme was proposed in (Cea, 2006) by just using a second order discretisation for the two unit discharge components, whilst keeping a first order discretisation for the water depth. In such a way the numerical diffusion is much reduced, without a significant reduction on the numerical stability of the scheme. In order to avoid spurious oscillations of the free surface when the bathymetry is irregular, an upwind discretisation of the bed slope source term is used. This has proved to be more stable and accurate than a centred scheme (Bermudez, 1998). Bed friction, rainfall and infiltration are discretised with a centred semi-implicit scheme at the cell nodes. The numerical scheme is explicit in time, so the CFL restriction applies over the time step.

Some of the main numerical difficulties of applying shallow water models to rainfall runoff prediction are the presence of highly unsteady wet-dry fronts, the extremely small water depths, and the high bed friction stresses which must be computed (in many regions of the spatial domain the water depth is of the order of millimetres or centimetres). All these issues may cause numerical instabilities and lack of accuracy if the numerical schemes used to solve the shallow water equations are not robust and accurate. Another desirable property of the numerical scheme is the conservation of mass, which means that no water is lost or gained during the computation due to numerical errors. Although this might seem obvious, not all the schemes guarantee the conservation of mass, especially in the presence of unsteady wet-dry fronts with very small water depths, as it is the case in the applications studied in this paper. In addition of being conservative, finite-volume schemes have proved to be very robust and accurate for the modelling of shallow water flows with wet-dry fronts, and are therefore especially suitable for the simulation of flood events generated by rainfall runoff.

3. EXPERIMENTAL VALIDATION

An experimental validation of the model in simple laboratory geometries was undertaken before application of the model to more complex real watersheds. The results obtained in two rainfall runoff validation tests are presented here. In the first validation test the experiments done by Iwagaki (1955) in a one-dimensional channel with three different slopes are modelled. The second validation test models some new rainfall runoff experiments over a 2D simple geometry, which have been designed for the validation of runoff numerical models. Those experiments are still being undertaken at the present time, but some first results are presented in this paper.

3.1. 1D Validation test: 1D rainfall runoff in a three-slope channel

In this validation test the numerical model was used to compute the hydrograph generated by a non-uniform rainfall intensity in a three-plane cascade surface. The experimental results of Iwagaki (1955) were used to compare with the numerical predictions.

The experiments of Iwagaki were conducted in a 24m long laboratory flume made of very smooth aluminium. The flume was divided into three reaches of equal length (8m) but different slope (0.020, 0.015 and 0.010 respectively from upstream to downstream). The rainfall intensity over the upper, middle and lower reaches was respectively 3890, 2300 and 2880 mm/h (from upstream to downstream). The duration of the rainfall for three different experimental cases were 10, 20 and 30s. This flow conditions produce rapidly varying flow, since the highest rainfall intensity occurs in the upstream steepest reach, while the lowest rainfall intensity and bed slope are those of the downstream reach.

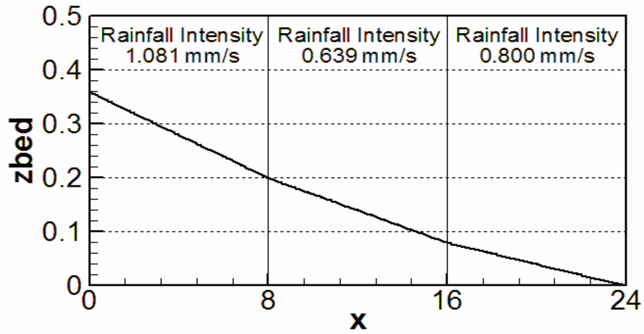

Figure 1. 1D validation test. Longitudinal profile.

The mesh size used in the numerical simulations was Δx=0.1m (240 elements along the 24m length experimental channel). Since an explicit discretisation in time is used in the numerical solver, the time step is restricted by the CFL condition. All the results presented in this section have been obtained with the first order upwind Roe scheme.

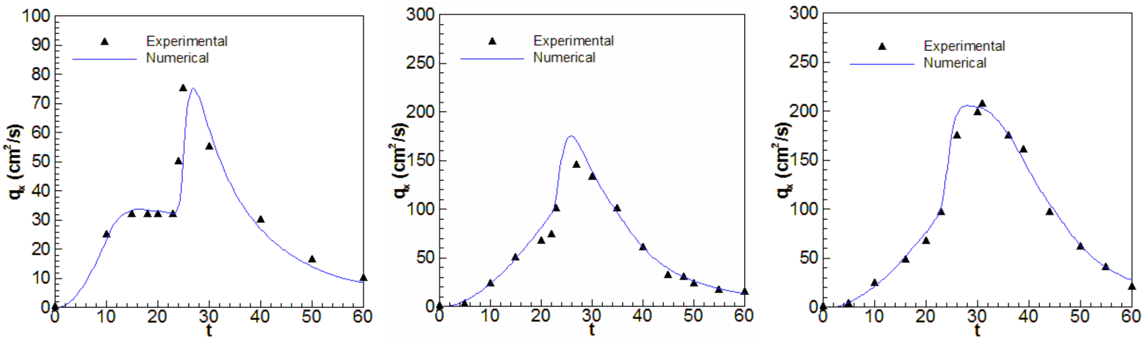

Figure 2. Numerical and experimental hydrograph for the 1D validation test. Case 1 (left), Case 2 (middle) and Case 3 (right).

The calibration of the bed friction coefficient for this validation test was done with the experimental results of case 1. The Manning coefficient which gives a best fit between experimental and numerical results for that case is n=0.009$s \cdot m^{-1/3}$, which is a sensible value for the smooth surface used in the experiments of Iwagaki. This bed friction coefficient, obtained from the calibration of case 1, was maintained in the numerical simulation of cases 2 and 3, producing also very good agreements with the experimental data (Figure 2).

In this case the numerical results are quite sensitive to the bed friction coefficient, as it is shown in Figure 3. If the bed friction is too large the peak discharge is underestimated, while a too low value of Manning coefficient overestimates noticeably the peak discharge (Figure 4 and Table 1).

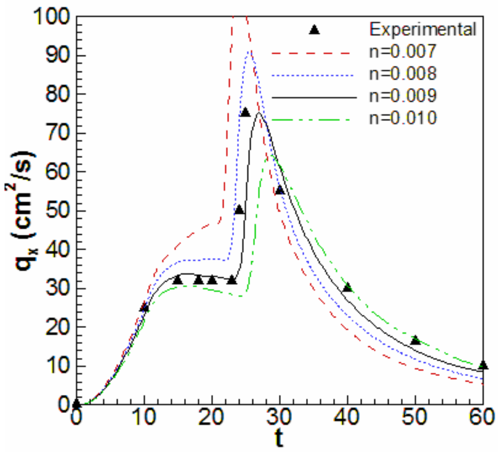

Figure 3. Sensitivity of the numerical results to the bed friction coefficient. 1D validation test. Case 1.

	n=0.006	n=0.007	n=0.008	n=0.009	n=0.010	Experimental
Peak discharge (cm^2/s)	128	107	91	75	64	73
Time peak (s)	23	24	25	27	29	26

Table 1. Sensitivity of the peak discharge to the bed friction coefficient. 1D validation test. Case 1.

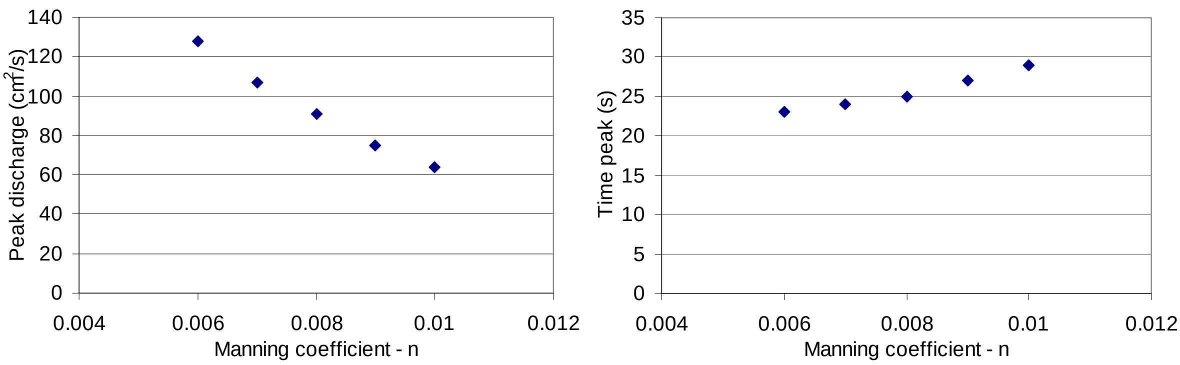

Figure 4. Sensitivity of the peak discharge to the bed friction coefficient. 1D validation test. Case 1.

3.2. 2D Validation test: 2D rainfall runoff

In this test the shallow water model is used to compute the hydrograph generated by a uniform rainfall over a simple 2D geometry. The numerical results are compared with experimental data obtained in a laboratory model specially designed for the validation of 2D runoff model. The results for three different rainfalls over the same geometry are presented in this paper. At the present time more experiments are being undertaken with different geometries and different surface roughness.

The geometry consists in a rectangular basin made of three planes of stainless-steel, each of them with a slope of 0.05 (Figure 5). The dimensions of the basin are 2m x 2.5m. There are two walls which increase the time of concentration of the basin and therefore, increase the length of the outlet hydrograph. Rainfall is simulated with a grid of nozzles (100 nozzles) distributed evenly over the basin. The only variable measured in the experiments is the outlet hydrograph, which will be used to validate the numerical results.

Three cases have been modelled, which correspond to three different rainfall intensities. In case C1 the rainfall intensity is 317mm/h during 45s. In case C2 the rainfall has an intensity of 320mm/h during 25s, it stops for 4s, and it starts again for 25s with the same intensity. Case C3 is similar to case C2, but the rainfall stops for 7s, and the intensity of the rain is 328mm/h.

The main difficulties which had to be faced during the experiments were four: (1) how to achieve a rainfall intensity spatially uniform, (2) how to diminish the surface tension of the water, which prevents the movement of water when the water depth is too small, (3) how to reduce the leaking of the nozzles when the rain is stopped, and (4) how to consider a small time lag which happens at the beginning of the experiments during which the intensity changes from zero to a constant value (it takes approximately 3-4 seconds to reach a constant intensity once the tap is opened). In cases C2 and C3, this time lag happens only at the beginning of the experiment, and not when the rainfall is stopped and started again after 4 or 7 seconds respectively. The influence of these four issues in the results is difficult to evaluate *a priori*, but in any case, all these sources of uncertainty should be taken in consideration in the experimental validation of the numerical results.

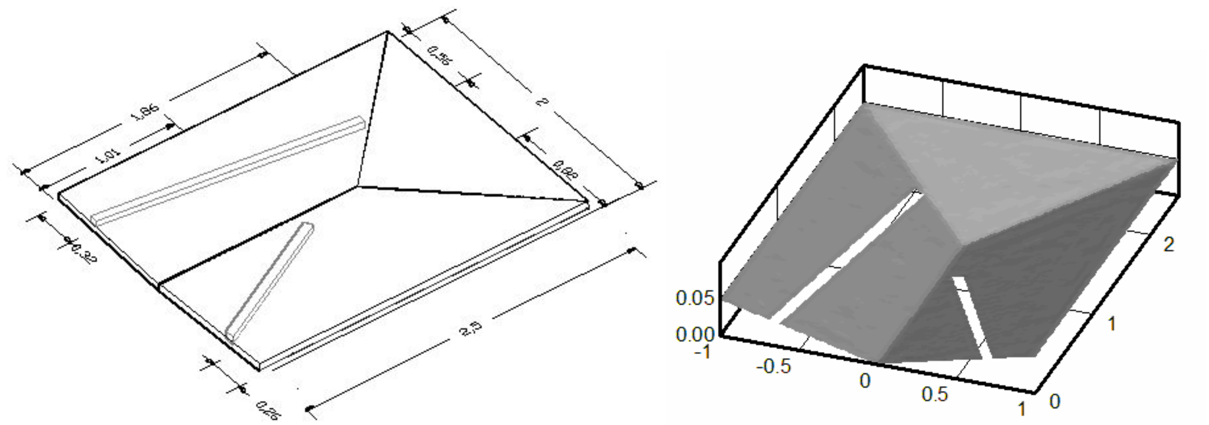

Figure 5. 2D validation test. Experimental setup (left) and 3D view of the numerical model (right).

The numerical mesh used to model the experiments is shown in Figure 6. The average element size of the mesh is 3.5cm^2 (15000 unstructured cells over a surface of 5m^2). Since the numerical solver uses an explicit time discretisation, the time step is restricted by the CFL condition.

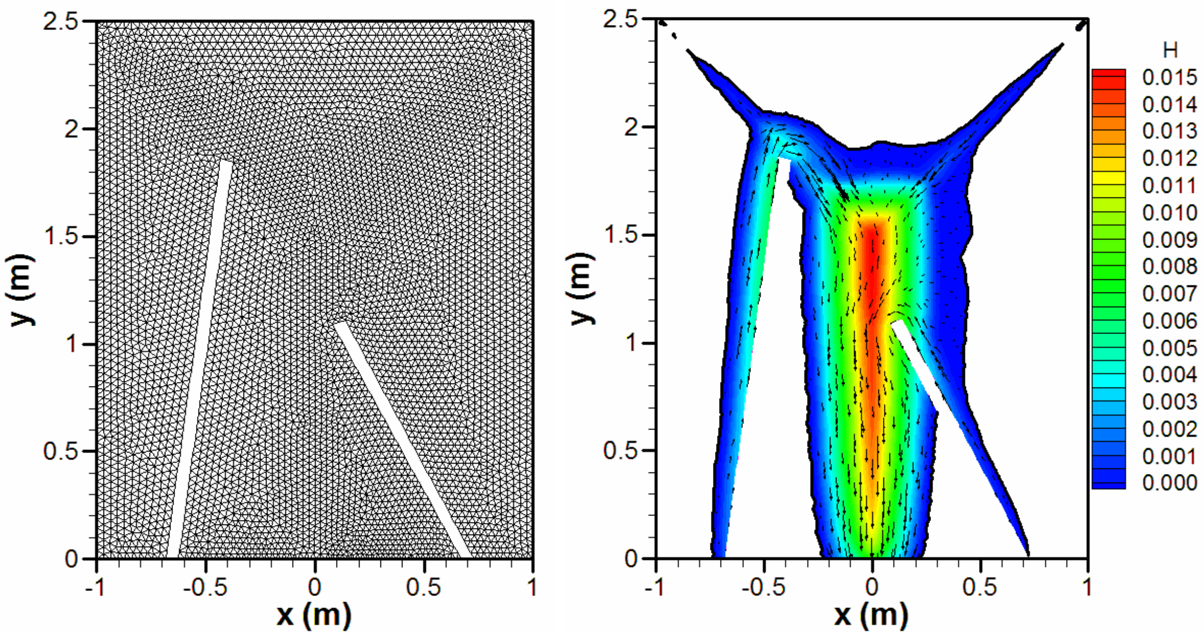

Figure 6. 2D Validation test. Case 1. Numerical mesh (left) and water depth and velocity field 5 seconds after the rain stops (T=50s). Regions with water depth smaller than 0.1mm in white.

Figure 7 shows a comparison between the numerical and experimental outlet hydrograph. Considering the different sources of uncertainty in the experimental results which have been mentioned above, the agreement is very satisfactory, especially for case C1. The shape of the hydrograph is very well predicted in all the cases, as well as the peak discharge. The tail of the hydrograph is slightly underestimated by the numerical model. This is probably due to the fact that, due to the water pressure, the nozzles used to generate the rainfall continue leaking for a while after the rain stops. This additional source of water, which is not taken into consideration in the numerical model, has the effect of increasing the outlet discharge in the tail of the hydrograph. In case C2 the first peak discharge occurs when the rainfall stops for the first time. At that point the discharge starts to decrease. After 4 seconds the rain starts again, but the discharge continues to decrease for approximately 6 seconds, because it takes some time until the new source of water reaches the outlet boundary. It should be noticed in Figure 7 the ability of the model to capture this process, as well as the second peak discharge.

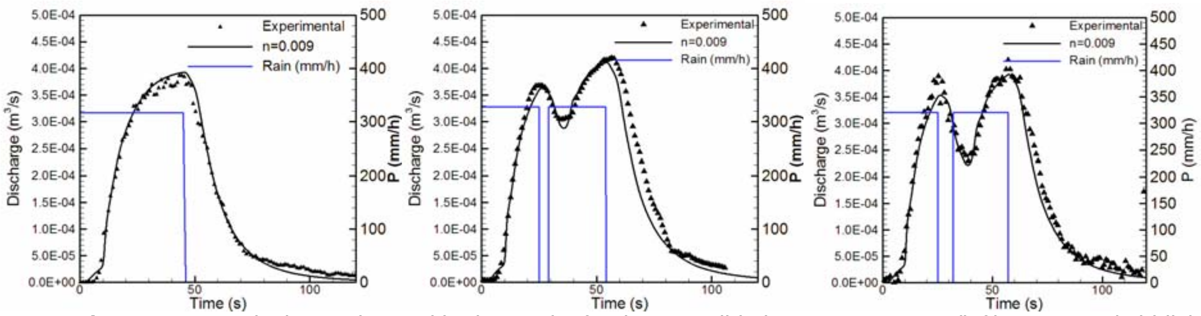

Figure 7. Numerical-experimental hydrographs for the 2D validation test. Case C1 (left), case C2 (middle) and case C3 (right).

4. APPLICATION TO A REAL WATERSHED

This section presents the application of the 2D shallow water model to a real watershed in the region of Galicia, Northwest of Spain. The watershed considered has a surface of approximately 5Km^2. The bed surface is mainly fractured granite, scrub and bushes. The terrain is very steep, being the mean slope approximately 35% (20 degrees).

At the present time field data for comparison with the numerical calculations are not available yet, but a field campaign is being undertaken in order to obtain a set of rainfall intensities and discharges which will be used to calibrate and validate the numerical model.

The numerical mesh for the watershed is shown in Figure 8. The mesh is rather coarse, being the average element size of the control volumes 1192m^2 (4111 unstructured cells over a surface of 4.9 Km^2). Since the numerical solver uses an explicit time discretisation, the time step is restricted by the CFL condition. The computations start with all the watershed dry as initial condition (zero water depth). The only boundary condition imposed is zero water depth gradient at the outlet boundary, which is located in the south part of the watershed, in the main section of the main river reach. In order to check the sensitivity to the bed friction coefficient, three different Manning coefficients have been used in the simulations n=0.05, 0.10, 0.50 (Figure 9). These values, specially the last one, might seem extremely high, but it should be noticed that, in most of the watershed, we are working with water depths of the order of centimetres, over a rather rough bed surface mainly made of rock and scrub. Therefore, in all these regions the Manning coefficient should be considered just as a calibration parameter which is used to characterise all the friction effects of micro-rugosity over the water runoff. It should be remarked that we have decided rather arbitrarily to use the Manning formula to characterise the friction effects. Nonetheless, other formulations could be used or proposed for this *quite unusual* application of the 2D shallow water equations.

Simulations with infiltration and with no infiltration have been done. The infiltration parameters for the Green-Ampt's formulation have been estimated from typical values in the region (Galicia, Northwest Spain) for a fractured granite soil. The following values have been considered: permeability k_s = 10-6m/s, porosity ϕ = 0.01, and suction ψ = 0.1m. Due to the high intensity and short duration of the storms considered (see hietogram in Figure 9), other mass losses as evapotranspiration, interception by vegetation and soil retention are not significant in this case, and do not need to be considered in the computations. In any case, all these losses would take place at the beginning of the hietogram, and would not affect the peak discharge.

With the mesh used in this case and the rainfall intensities, the model runs very fast (it takes approximately 1 minute of computational time on a personal computer to compute 3 hours of real time). Therefore, with the present computer capabilities it could be possible to use these kind of models for real time fast-flood forecasting in small and medium catchments. Nonetheless, validation of the models with field data in watersheds of different size with different kind of terrain is still needed in order to verify the accuracy of the hydrographs computed.

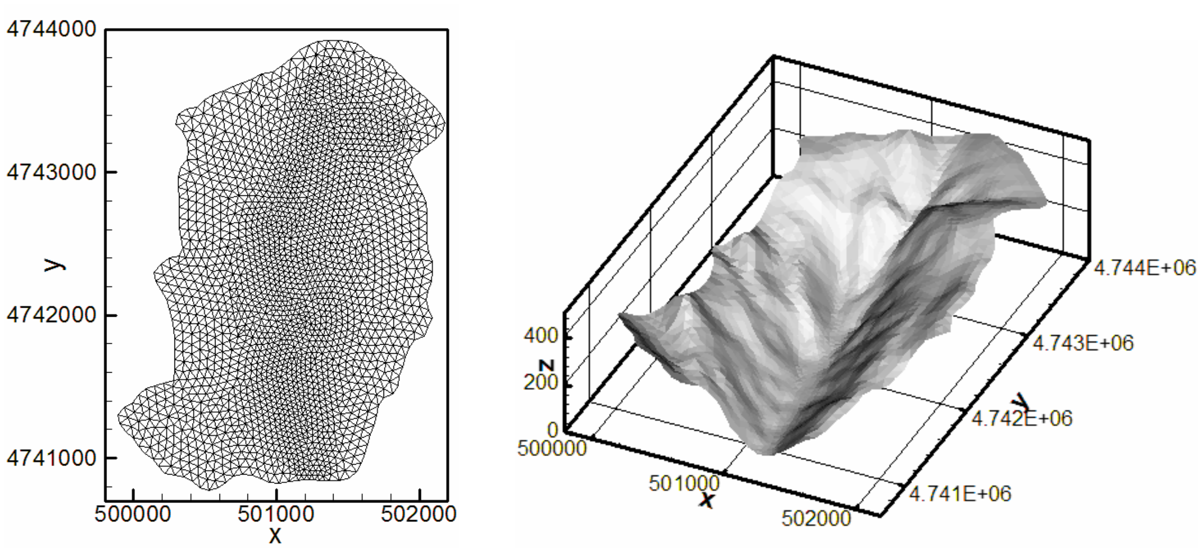

Figure 8. Numerical model of the watershed at Esteiro. Numerical mesh (left) and 3D view (right).

The sensitivity of the numerical results to the bed friction coefficient is high (Figure 9 and Table 2). Whereas infiltration diminishes slightly the peak discharge but does not change significantly the time of arrival, an increase in the bed friction coefficient diminishes considerably the peak discharge, delaying at the same time its time of arrival. The bed friction coefficient will be calibrated with experimental data from a field campaign which is being undertaken at the present time.

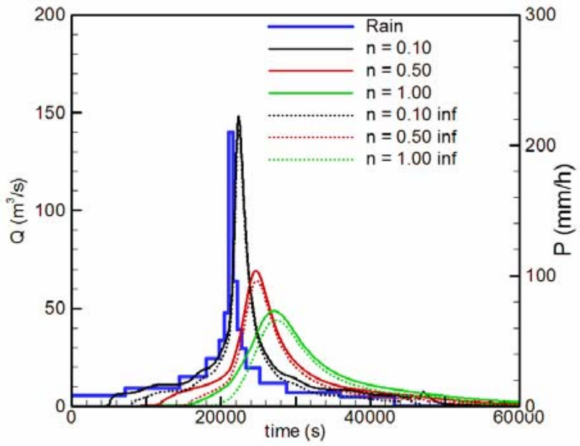

Figure 9. Hietogram for T=500 years, and hydrographs given by the numerical model for different bed friction coefficients, with infiltration and without infiltration.

	Peak discharge (m^3/s)			**Time peak** (h)		
	n=0.10	**n=0.50**	**n=1.00**	**n=0.10**	**n=0.50**	**n=1.00**
No infiltration	148	69	49	6.2	6.9	7.5
Infiltration	142	64	44	6.2	6.9	7.6

Table 2. Hietogram for T=500 years, and hydrographs given by the numerical model for different bed friction coefficients, with infiltration and without infiltration

One advantage of using 2D shallow water models in rainfall-runoff prediction is that they give us not only the hydrograph at the watershed outlet, but also detailed information of the flood extension, including flow velocities and water depth, as well as its evolution in time (Figure 10).

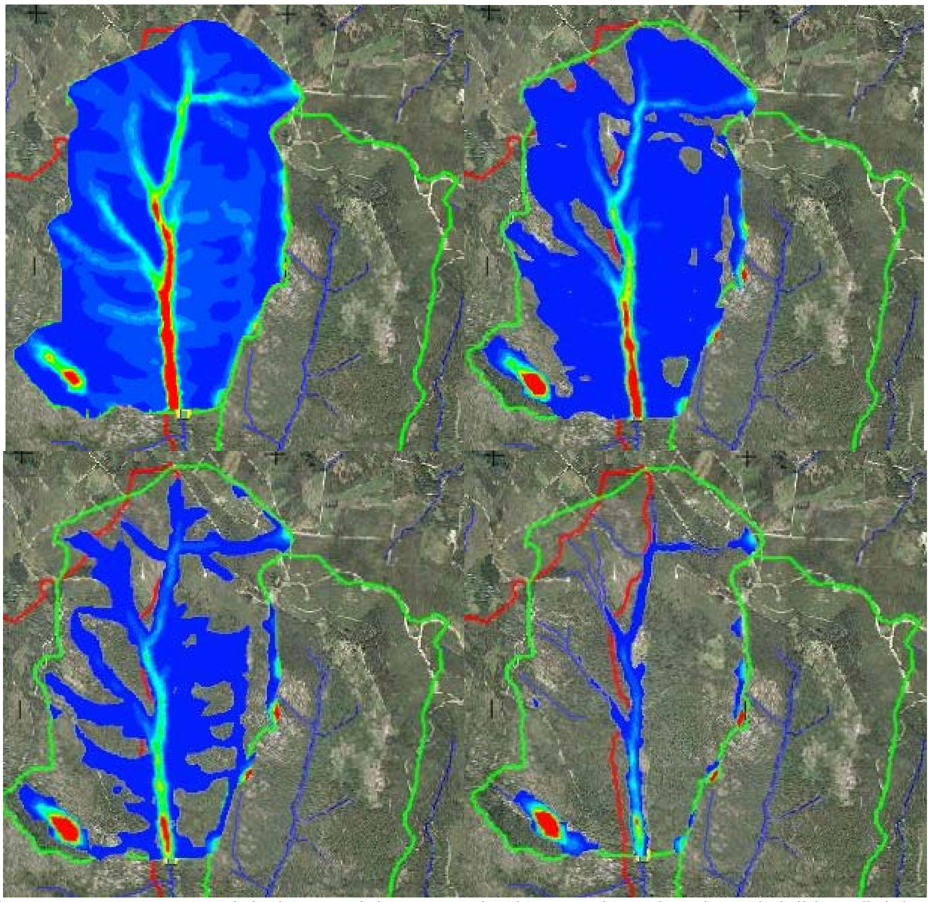

Figure 10. Sequence of drainage of the watershed at Esteiro, after the rainfall has finished.

5. CONCLUSIONS

This paper presents the preliminary results of a research line in which it is being studied the application of two-dimensional shallow water models to simulate rainfall runoff, with the aim of forecasting fast flood events in small catchments. Some preliminary results concerning the experimental validation of the model as well as its application to a real watershed have been discussed, and some new rainfall runoff laboratory results have been presented, which might be used by other researchers in order to validate their numerical models and predictions.

The experimental validation of the model in 1D and 2D simple laboratory geometries is encouraging. The numerical-experimental agreement is very satisfactory, with just one calibration parameter, the Manning's coefficient. In addition, the Manning's coefficient which best fits the experimental data is the expected value for the smooth bed surface used in the experiments.

The application of the model to a real watershed of approximately $5Km^2$ is feasible from a numerical point of view, considering model stability and computational time. Field measurements of rainfall intensities and water discharge are being undertaken in the Esteiro watershed in order to validate the numerical predictions. Unfortunately, no experimental results are available yet. Nonetheless, the numerical computations are stable and fast enough in order to permit the application of shallow water models to real time forecasting and management of fast flood events in complex watersheds.

No matter which bed friction formulation is used (Manning, Keulegan, Chezy or any other), the bed friction coefficient in this kind of application should be considered just as a calibration parameter which is used to characterise all the friction effects of micro-rugosity over the water runoff. Other bed friction formulations different from those used commonly in river engineering might be proposed for this *quite unusual* application of the 2D shallow water equations.

In any case, further validation of 2D shallow water models applied to rainfall-runoff simulation with field and laboratory data is still needed, in order to verify the accuracy of the results and to calibrate the bed friction parameter.

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