Deficit irrigation reliability analysis: application of constraint state formulation and AFOSM

M. Shekarrizfard [b], and A. Ganji [♣a]

[b] Water Engineering Department, Agricultural College, Shiraz University, Shiraz, Iran
[a] Department of Desert Region Management, Shiraz University, Iran

Abstract

In the present study, two types of uncertainty namely demand side uncertainty and allocation side uncertainty are considered. A constraint state formulation for stochastic optimization of weekly irrigation strategy is used to incorporate the effects of the above uncertainties in deficit irrigation scheduling. This formulation is based on the first and second order moment analysis of the stochastic soil moisture state variable, considering soil moisture at both saturation and deficit cases as the maximum and minimum bounds, respectively. As a result, mean and variance of actual evapotranspiration are used for reliability analysis of the relative net benefit based on the Advanced First Order Second Method (AFOSM) for deficit irrigation case. This method is widly used in engineering applications. The optimization and simulation results are indicative of the importance of crop water demand uncertainty consideration when determining optimal deficit irrigation strategy. Also the probability of intra-seasonal crop water stress index is determined based on the moment analysis and using the double bounded density function methodology. The results indicate that achieving a high long-term expected relative net benefit by decreasing the crop water allocation and increasing the irrigated land may fail as a strategy when crop demand uncertainty is ignored.

Introduction

Rainfall is the main source of uncertainty in arid and semi-arid areas that affects irrigation scheduling due to its large spatial and temporal variations (Heermann et al., 1990; Sunantara and

♣ Corresponding author Email : ganji@shirazu.ac.ir Tel: +98-511-5029956

Ramfrez, 1997). Ramirez and Bras (1985) showed the importance of the stochastic of rainfall input in the context of a single crop, using direct stochastic dynamic programming for optimal irrigation scheduling. In response to the uncertainty in deficit irrigation scheduling, the random nature of parameters has been considered in two sides of a crop water management problem, crop water demands calculation and the allocation policy establishment. Considering these randomness, research on irrigation scheduling issues can be divided into three different groups, i.e., deterministic model (Mannocchi and Mecarelli, 1994; and Kipkorir and Raes, 2002), demand side random based models (Yoo et al. 2005; Ganji et al. 2006a, and 2006b) and allocation side random based-models (Ganji et al. 2006c). The previous work by Ganji et al. (2006a) dealt with deficit irrigation, where by the papers by Ganji et al. 2006 b, and c were based on full-irrigation condition. Both types of randomness are ignored by deterministic models, and as a result the estimated relative net benefit and the proposed irrigation may fail especially in the deficit irrigation case (Ganji et al. 2006b). Similar researches on deterministic soil moisture dynamic are Zhang et al. (2004) and Chen et al. (2004), which can be improved by considering the randomness when using the methodologies developed by Rodriguez-Itrube et al. (1999).

As an example for the demand side random based models, Yoo et al. (2005) developed a simple zero-dimensional soil moisture dynamics model along with the generated rainfall from a rectangular pulse Poisson process. Basically, first- and second-order statistics of soil moisture are derived analytically for both instantaneous and locally averaged cases, which are then used to evaluate the effect of rainfall on the soil moisture statistics. A notable work in stochastic irrigation scheduling is presented by Villalobos and Fereres (1989). They developed a simulation model by coupling a simple daily rainfall generator to a water balance model that determines irrigation dates and amounts. As a single work on the allocation side random based model, Ganji et al. (2006c) modified the model developed by Ganji et al. (2006b) to incorporate a random allocation policy instead of the determinestic case for full irrigation scheduling. However, in the case of deficit irrigation, considering both types of uncertainty (demand side and allocation side), which may affect the crop yield seriously by water stress (English,

1981) has not been reported to date. Deficit irrigation increases the crop sensitivity to any small water stress, intensifying the role of demand uncertainty and randomness in cropping benefit. Actually any inaccurate forecasting about the required demand will increase the risk of loss (English, 1981). English et al. (1990) reported that only where there are constraints on capital, energy, labor or other essential resources, or when costs of any of these resources are particularly high, deficit irrigation can be used as a strategy to increase profit. Nevertheless, even in the absence of the features described in English et al. (1990), some researchers attempted to predict the crop yields and benefit for deficit irrigation, ignoring inherent crop water demand uncertainties. By considering the uncertainty, reliability analysis of the relative net benefit which can highly be affected by water stress, seems to be necessary.

In response to the above circumstances and also requisiteness to reliability analysis of the relative net benefit in deficit irrigation case, and continuing on the work by Ganji et al. (2006a), a new modeling framework is developed based on the constraint state formulation and Advanced First Order Second Method (AFOSM) to consider both types of uncertainty in modeling. For this purpose the formulation by Ganji et al. (2006a) has been modified to consider the uncertainty in allocation and also the second moment of actual evapotranspiration is developed to determine the second moment of the actual evapotranspiration as required for reliability analysis. Then, the moments of the soil moisture and actual evapotranspiration are applied for the reliability analysis of the relative net benefit in the deficit irrigation case, based on the AFOSM methodology. The First Order Second Moment (FOSM) method and the Advanced First Order Second method (AFOSM) were originally developed in the past two decades to assess the safety of structural components and systems (Madsen et al. 1986; Melchers 1987).

Also, the weekly water stress probability density functions were determined by moment analysis, based on the proposed formulation for crop water stress as shown by Rodriguez-Itrube (1999a) and applying the Kumaraswamy distribution (Kumaraswamy 1980). Kumaraswamy distribution is a Double-Bounded probability density function especially suitable for environmental variables that are physically bounded. To verify the result of the optimization model, a simulation model is proposed which shows the results obtained from optimization using the new stochastic models compared favorably with those obtained from simulation. In the presence of both types of demand and allocation sides randomness, reliability analysis of relative net benefit shows that deficit irrigation may not increase the relative net benefit when costs of any of lateral resources (i.e. labor cost, system operation and maintenance and, etc) are particularly high. As an outline for this research, the seasonal crop-water production function by Jensen (1968) is discussed. Then the first and the second moments of soil moisture and first moment of actual evapotranspiration are presented, as developed by Ganji et al. (2006c). In the following, a formulation for the second moment of the actual evapotranspiration is developed which will be used in AFOSM analysis of the relative net benefit in deficit irrigation case. Also the moments of a proposed crop stress index are developed based on the Double-bounded density function method. Finally, AFOSM is discussed in detail and the overall structure of the optimization model is presented briefly

Moment analysis

First and Second moments of actual evapotranspiration

To deal with the uncertainty in calculation of the relative yield and crop water demand, Ganji et al. (2006b) developed a stochastic optimization framework to incorporate the crop demand uncertainty in the irrigation scheduling problem. Their proposed method is modified to consider the randomness in irrigation policy by Ganji et al. (2006c). Their proposed methods are extended to estimate the ET's second moment in this paper, considering the randomness in irrigation policy for deficit irrigation. Evapotranspiration is a function of current soil moisture conditions and can be represented as:

$$ET_t = \begin{cases} 0 & \theta_t \leq \theta_{pwp} \\ \dfrac{ET_{p_t}(\theta_t - \theta_{pwp})}{(1-p)(\theta_{FC} - \theta_{pwp})} & \theta_{pwp} \leq \theta_t < (1-p)(\theta_{FC} - \theta_{pwp}) \\ ET_{p_t} & \theta_t \geq (1-p)(\theta_{FC} - \theta_{pwp}) \end{cases} \quad \text{.................................} (1)$$

where θ_t is the relative soil moisture at the beginning of a weekly period t, ET_t is the long-term average of weekly actual evapotranspiration for a specific plant, L_t is the long-term average of weekly leaching fraction, and η_t is the random noise term of soil moisture balance equation, θ_{FC} is the relative soil moisture at field capacity and θ_{pwp} is the relative soil moisture unavailable for plant growth or the water content of wilting point. Equation (1) can be represented using the indicator function as follows:

$$ET_t = ET_{p_t} . 1_{(\theta_t \geq (1-p)(\theta_{FC}-\theta_{pwp}))}(\theta_t) + \frac{ET_{p_t}(\theta_t - \theta_{pwp})}{(1-p)(\theta_{FC}-\theta_{pwp})} . 1_{(\theta_{pwp} \leq \theta_t \leq (1-p)(\theta_{FC}-\theta_{pwp}))}(\theta_t) \quad (2)$$

where n is the soil porosity, z_t is the root zone depth at time t, θ_t is the relative soil moisture at the beginning of a weekly period t *and* (1-p) is the function of total available soil water that can be depleted from the root zone before stress (reduction in ET) occurs. The expected value and variance of the evapotranspiration is also determined utilizing the first order Taylor series approximation as developed by Ganji et al. (2006c). As previously mentioned, since the variance of actual evapotranspiration is required for the AFOSM reliability analysis, a new formulation is developed based on the first order Taylor series approximation (see Eq. 3).

$$E(ET_t)^2 = \frac{ET_{p_t}^2}{2}\left(1 - erf(\frac{A max}{\sqrt{2Var\eta_t}})\right) + \left(\frac{ET_{p_t}}{nz_t(1-p)(\theta_{FC}-\theta_{pwp})}\right)^2 \left\{ \frac{(nz_t\theta_{max}^t - A max)^2}{2} \right.$$

$$(erf(\frac{A max}{\sqrt{2Var\eta_t}}) - erf(\frac{A min}{\sqrt{2Var\eta_t}})) - \frac{1}{2}\sqrt{\frac{Var(\eta_t)}{2\pi}} \left\{ 2A max * exp(-\frac{1}{2}\frac{(A max)^2}{Var\eta_t}) - \sqrt{2\pi Var\eta_t} \right.$$

$$\left. erf(\frac{A max}{\sqrt{2Var\eta_t}}) \right\} - \frac{1}{2}\sqrt{\frac{Var(\eta_t)}{2\pi}} \left\{ 2A min * exp(-\frac{1}{2}\frac{(A min)^2}{Var\eta_t}) - \sqrt{2\pi Var\eta_t}\ erf(\frac{A min}{\sqrt{2Var\eta_t}}) \right\} +$$

$$\frac{(nz_t\theta_{min}^t)^2}{2}(erf(\frac{A max}{\sqrt{2Var\eta_t}}) - erf(\frac{A min}{\sqrt{2Var\eta_t}})) - nz_t\theta_{min}^t(nz_t\theta_{min}^t - A min) \left\{ erf(\frac{A max}{\sqrt{2Var\eta_t}}) - \right.$$

$$\left. \left. erf(\frac{A min}{\sqrt{2Var\eta_t}}) \right\} - \sqrt{\frac{Var(\eta_t)}{2\pi}} \left\{ exp(-\frac{1}{2}\frac{(A max)^2}{Var\eta_t}) - exp(-\frac{1}{2}\frac{(A min)^2}{Var\eta_t}) \right\} \right\} \quad (3)$$

where η_t is the random noise term of soil moisture balance equation, $\theta_t^{\min}$ is the lowest level of soil moisture state variable which is determined based on the allowable level of irrigation deficit for a plant, and $\theta_t^{\max}$ is the maximum allowable soil moisture.

$$A = (k_t + Ra_t + n(z_t - z_{t-1})\theta_r - ET_t) \qquad (4)$$

$$A\,max = nz_t\theta_t^{max} - (k_t + Ra_t + n(z_t - z_{t-1})\theta_r - ET_t) \qquad (5)$$

$$A\,min = nz_t\theta_t^{min} - (k_t + Ra_t + n(z_t - z_{t-1})\theta_r - ET_t) \qquad (6)$$

$$B\,max = \left(nz_t\theta_t^{max} - n(z_t - z_{t-1})\theta_r\right) \qquad (7)$$

$$B\,min = \left(nz_t\theta_t^{min} - n(z_t - z_{t-1})\theta_r\right) \qquad (8)$$

Moments of crop water stress

Porporato, et al. (2001) attempted quantification of plant water stress related to the soil moisture conditions. They assumed that the static stress ζ is zero when soil moisture is above the level of incipient stomatal closure, s*, and reaches a maximum value equal to one when soil moisture is at the level of complete stoma closure (wilting), i.e., $\zeta = 0,\ for\ \ s > s*$ and $\zeta = 1\ \ for\ s < s_w$. A reasonable general form for the static stress can thus be taken as:

$$\zeta(t) = \left[\frac{s* - s(t)}{s* - s_w}\right]^d, \qquad for \qquad s_w \le s(t) \le s* \qquad (9)$$

where d is a measure of nonlinearity of the effects of soil moisture deficit on plant conditions (Rodriguez-Itrube 1999a,b). The simple relationship between ζ and "s" allows one to develop equations for the expected value and variance of ζ based on the moments of the soil moisture (see Ganji et al., 2006a, b). Using the second-order Taylor series approximation method expectations of equations for water stress's first and second moments can be developed as follows:

$$E(\zeta) = (\frac{\theta_t^{max} - E(\theta_t)}{\theta_t^{max} - \theta_t^{min}}) + \frac{1}{2} E(\theta_t)^2 (\frac{\theta_t^{max} - E(\theta_t)}{\theta_t^{max} - \theta_t^{min}})^{d-2} \frac{d(d-1)}{(E(\theta_t) - \theta_t^{min})^2} \quad (10)$$

$$E(\zeta)^2 = (\frac{\theta_t^{max} - E(\theta_t)}{\theta_t^{max} - \theta_t^{min}})^{2d} + E(\theta_t)^2 (\frac{\theta_t^{max} - E(\theta_t)}{\theta_t^{max} - \theta_t^{min}})^{2d-2} \frac{d(2d-1)}{(E(\theta_t) - \theta_t^{min})^2} \quad (11)$$

Also, the weekly water stress probability density functions were determined using the resulting moments (see Ganji et al., 2006a, b). Using the second-order Taylor series approximation method expectations of equations for water stress's) and applying the Kumaraswamy distribution (Kumaraswamy, 1980), which is a Double-Bounded probability density function especially suitable for environmental variables that are physically bounded.

Fitting the double-bounded density function

The soil moisture state density function is hybrid in nature, with spikes at maximum and minimum soil moisture capacities (bounds), representing the probability of saturation and soil moisture deficit. Considering equation (9) this property is transformed to the proposed stress index, and as a result should be considered as a double-bounded random process. Kumaraswamy (1980) developed a generalized probability density function for such a double-bounded random process. His proposed formulation was presented for the case of non-zero probability at the lower bound and zero probability at the upper bound of the random variable, called DB-CDF. Fletcher and Ponnambalam (1996) extend the DB-CDF that caters for non-zero probability at the maximum storage bound. Considering their modified methodology, the DB-CDF for the proposed soil moisture based stress index can be developed as:

$$F(\varepsilon) = P_{st} + (1 - [P_{st} + P_{sa}])[1 - (1 - \zeta^a)^b] + \int_0^x P_{sa} \delta(u-1) du \quad (12)$$

where δ is the Direct delta function and a and b are assumed the positive coefficients. P_{st} is the probability of the lower bound soil moisture violation which can be determined based on the third term in the right hand side of the first moment of soil moisture equation as developed by Ganji et al. (2006b).

P_{sa} is the probability of the upper bound soil moisture violation which can be determined based on the fourth term in the right hand side of the first moment of soil moisture equation. By differentiating (12) with respect to ζ, the double-bounded probability density function is as:

$$f(\varepsilon) = (1 - [P_{st} + P_{sa}])ab\zeta^{a-1}(1-\zeta^{a})^{b-1} \quad 0 < \zeta < 1 \tag{13}$$

According to expressions given by Kumaraswamy (1980) and Fletcher and Ponnambalam (1996), the n[th] moment of the double-bounded soil moisture-based stress index variable is given as:

$$E(\varepsilon)^{n} = (1 - [P_{st} + P_{sa}])\frac{(\frac{n}{a})!b!}{(\frac{n}{a}+b)!} + P_{sa} \tag{14}$$

where ! denotes factorial. The mean and the variance of the proposed index is derived as (n=1 and 2):

$$E(\varepsilon) = (1 - [P_{st} + P_{sa}])\frac{(\frac{1}{a})!b!}{(\frac{1}{a}+b)!} + P_{sa} \tag{15}$$

$$Var(\varepsilon) = (1 - [P_{st} + P_{sa}])\frac{(\frac{2}{a})!b!}{(\frac{2}{a}+b)!} + P_{sa} - (E(x))^{2} \tag{16}$$

The right hand side of equation (15) and (16) is determined during optimization which allows for the solutions of the parameters a and b. After determining a and b, the probability density function of ξ can be determined using the equation (13).

Advanced First Order Second Method (AFOSM)

In optimal irrigation scheduling problem for a plant, the uncertain variable of interest can be considered as the actual evapotranspiration (*ET*) at any specific duration of the plant growing. Irrigation scheduling-related parameters can include mean and the variance of soil moisture content, rainfall mean and variance, soil and plant properties, and irrigation depth to calculate the required statistical information about ET. The soil moisture parameters and the resulting actual evapotranspiration values can be considered as uncertain. The AFOSM technique is used to estimate the relative net benefit of a deficit irrigation strategy using soil moisture balance equation as presented Ganji et al. (2006b). This method takes the first two statistical moments of a linear approximation of the performance function and attempts to find the minimal distance from the given nominal point to the tangent hyper-plane. This distance provides a measure of the crop water production reliability. In this paper the interest is in assessing the potential of relative net benefit as a result of the deficit irrigation. Details of AFOSM method can be found in Seifi et al. (1999).

Overall optimization framework

The relative net benefit as developed based on the crop water production function should be maximized with respect to the first and second moment of soil moisture and the first moment of actual evapotranspiration (Ganji et al., 2006c) in addition to equation (3) to determine the optimal irrigation policy as well as the required statistical properties for AFOSM reliability analysis. Additional constraints include restriction of irrigation amount to the probable maximum available soil moisture capacity of each stage (t) as in equation (17). Due to the variability in available soil moisture capacity, it is considered as a chance constraint as follows:

$$Ir_t \le nz_t\theta_t^{max} - (Ra_t + n(z_t - z_{t-1})\theta_r - \sqrt{2Var\eta_t} * erfinv(2p-1) - ET_t) - nz_{t-1}\theta_{t-1} \qquad (17)$$

where p is the probability of not violating the available soil moisture capacity as assigned by the decision maker. Lower value of p leads to higher permission of soil moisture capacity violation, resulting in a higher possibility of water loss. Restriction of the actual evapotranspiration to a maximum potential evapotranspiration, and restriction of irrigation depths to positive values are the other constraints that should be considered (equations 18 and 19).

$$ET_t \le ET_p \qquad (18)$$

$$k_t - nz_{t-1}\theta_{t-1} \geq 0 \qquad (19)$$

Materials and methods

In this paper, the usefulness of the formulation for the dynamics of the soil moisture presented in the previous parts of this paper is examined in its application to the winter wheat deficit irrigation scheduling problem. Required information and data are collected near an agricultural meteorological station at Badjgah, located in Fars province, south of Iran. Badjgah has a semi-arid climate, with average annual rainfall of 404 mm, most of which occurs during winter and spring. Annual potential evaporation is 1800 mm. Rainfall and evapotranspiration (using the FAO-56) data for the period 1983-2001 were used for the analysis carried out in this study. The daily rainfall record was extended using the Markov chain and Gamma distribution and the results are used to calculate the weekly mean and standard deviation of the rainfall. The calculations procedure presented by Tsakiris (1982) and Kipkorir and Raes (2002) is used to determine the weekly winter wheat sensitivity indices utilizing the seasonal sensitivity indices. The proposed model results compare favorably with the results obtained from simulation under real world conditions.

Results and Discussions

In this paper the modified approach, as developed by Ganji et al (2006c), is applied to determine the possibility of deficit irrigation, considering both demand and allocation uncertainties in deficit irrigation scheduling optimization. For this purpose, an irrigation policy is defined using a random policy and the second moment of the actual evapotranspiration is developed based on it. Then the result of the optimization model is used for reliability analysis of the relative net benefit for the deficit irrigation case. Also a methodology is proposed to estimate the probability of the weekly crop water stress index.

To verify the proposed optimization model a simulation model is developed. For this purpose the resulted irrigation strategy (from the optimization model) is used with fixed input values for the simulation model. The simulation model is mainly based on a simple soil water continuity equation, which uses the random generated rainfall and crop characteristics as inputs. As the first step for the verification of the model, the simulated mean soil moisture and standard deviation are compared with the corresponding optimized values. Figure 1(a) shows the results of comparison by a correlation coefficient of 99% for both simulated cases. Considering the randomness in irrigation policy improves the result of the soil moisture modeling for both of mean and variance of soil moisture values in respect to Ganji et al. (2006a). Also the actual evapotranspiration as resulted from the simulation and optimization compare quite well, as shown in Figure 1(b). The mean actual evapotranspiration is fitted to the mean simulated values with a 95% safety interval of the simulated actual evapotranspiration (see Figure 1b). The actual evapotranspiration variance is also determined based on the developed moment of the actual evapotranspiration for different weeks during the growing season in this paper, which is compared with simulation results in Figure 2. The above verification results show that the model works well to determine the first and the second moments of the soil moisture and actual evapotranspiration.

The reliability of optimal irrigation scheduling is determined based on the results of the proposed model (see Figure 3). The reliability is the complimentary factor of the failure and is equal to the number of not-violating soil moisture constraints (maximum soil moisture capacity and minimum critical soil moisture for a crop) for a specific time interval. The optimal irrigation reliability can be determined using the second and third parameters in the right hand side of the first moment of evapotranspiration (Ganji et al. 2007c), which respectively show the probability of the runoff, and deficit in soil moisture content (soil moisture less than specific threshold). The results of optimization model show that possibility of failure is low for the proposed optimal irrigation scheduling. After verification of the model, the soil moisture moments are used to estimate the moment for the proposed crop water stress index. This crop water stress index can be used to show the sensitivity of a crop to water stress after deficit treatment. For this purpose the double bounded density function analysis are used to determine the corresponding density function of the proposed index. The result of this analysis is shown in Figure 4 for four different growing periods of the winter wheat. As shown, the probability of crop water stress occurrence is high, which illustrates the high possibility of yield reduction due to any unpredictable water stress in weekly basis.

To explore the effect of the water stress on the final relative net benefit, AFOSM reliability analysis is applied based on the results of the optimization model. Figure 5 shows the AFOSM results for winter wheat. According to this figure, the achievement probability of more than 100 % relative net benefit, applying the deficit irrigation strategy and cropping area increasing, is about 0%. On the other hand, gaining a higher relative net benefit by decreasing the allocated water and increasing the available area is fails when the results of long term benefit analysis is explored.

Summary and Conclusion

Due to the inherent uncertainty in weather data, judgments about the possibility of achieving a higher relative net benefit by decreasing the allocated water (deficit irrigation) and increasing the cropping land may not be possible when a deterministic model is considered for analysis. As a continuation of the work by Ganji et al (2006a), allocation uncertainty is considered in deficit irrigation analysis, by utilizing a random irrigation policy in developing the second moment of the actual evapotranspiration instead of deterministic irrigation depth. The reliability analysis of the relative net benefit indicated using deficit irrigation strategy for the winter wheat can not increase the relative net benefit in long term. The results are also justified by moment analysis of a weekly crop-water stress index, which shows the high probability of crop stress in some weeks of the growing period.

Figures:

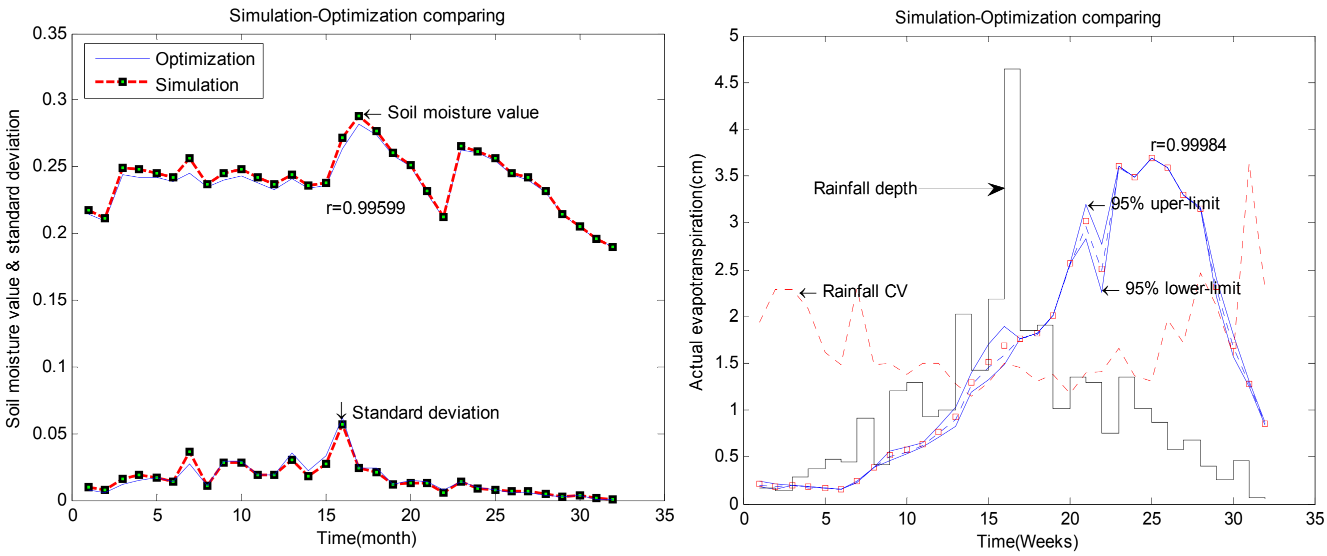

Figure 1. Comparison of mean and standard deviation of the soil moisture, and actual evapotranspiration for winter wheat as resulted from applied deficit irrigation strategy

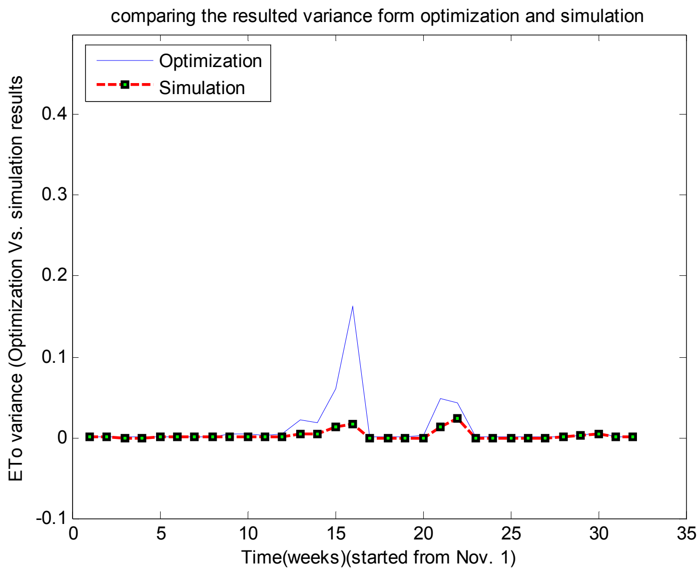

Figure 2. Variance of the actual evapotranspiration as resulted from optimization and simulation results

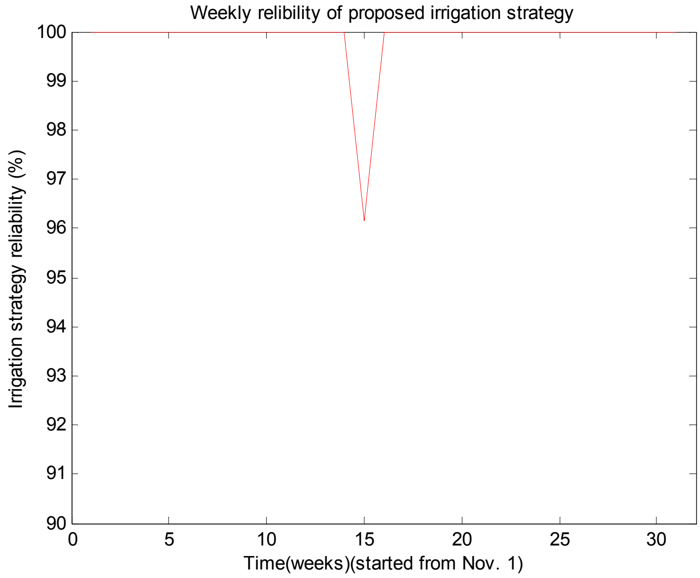

Figure 3. The weekly irrigation strategy reliability as resulted from optimization model

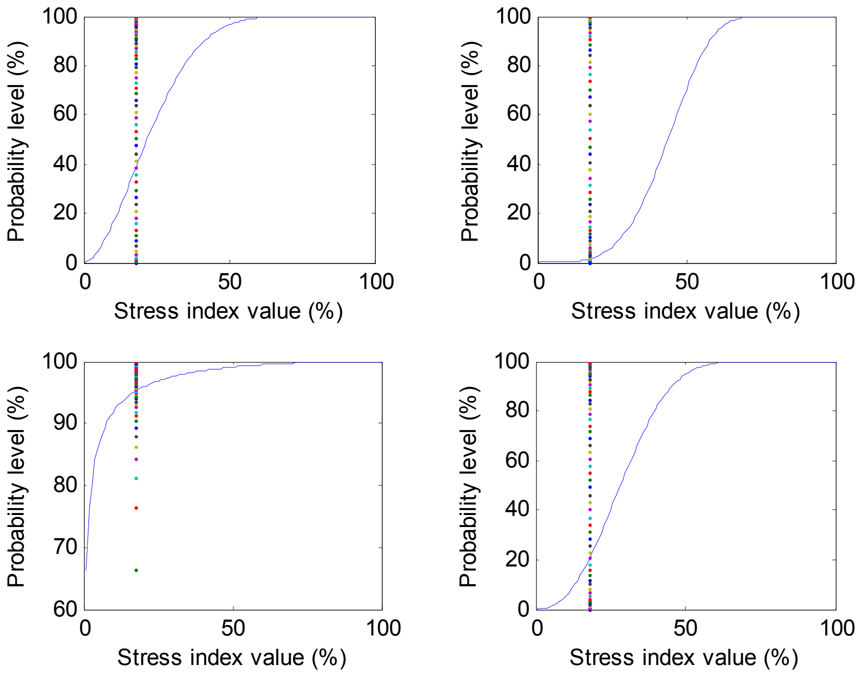

Figure 4. Cumulative probability function of the weekly water stress index as resulted from optimization model and DB-F analysis

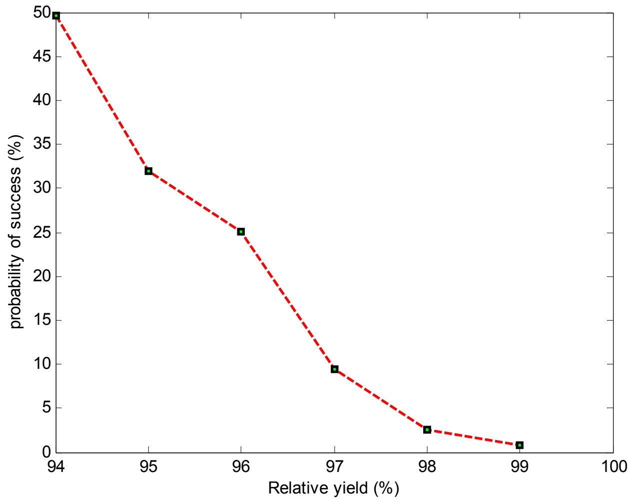

Figure 5. Cumulative probability function of the relative yield for deficit irrigation case as resulted from AFOSM reliability analysis

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