Water sharing among competing farmers in temperate climate: a study of different pricing mechanisms

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Abstract

The properties of a pricing rule, applied in an irrigation area in France, and of some of its derivatives, are studied through a formalized model, considering the Nash equilibria in a deterministic and in a stochastic environment. We show that when we add some freedom degrees in the pricing system, it is possible to limit the use of water for irrigation in temperate countries, to anticipate possible usage conflicts, to give assurance of budgetary equilibrium for the water user association, to incite the farmers to utilize less water, and to use the water in productions where its valorization is at its best. This method can be translated in tarification rules which can be made easy to understand, and econometric results show that the farmers reaction is conformable to what is expected, showing in passing the acceptability of this pricing rule.

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1 Introduction

Irrigation is one of the principal water uses in temperate countries, as in France. Limiting some consumption of this natural resource by the agricultural sector is therefore one of the more undisputed environmental problems. Moreover a better use of the water resource by farmers is now an explicit aim of the French Agriculture Ministry. Sharing a limited resource in order to optimize its use may be done using different tools (Perry, 2001), but it is all the more complicated in France where the resource legally does not belong to anybody. A lot of tools have been long documented in the literature, and sometime used, too much often without much success. But once recognized that "Water has an economic value in all its competing uses and should be recognized as an economic good" (Dublin declaration, 1992), the progresses in different fields of economic science may be used to diminish water "wastes" (Briscoe, 1996, Savenije et van der Zaag, 2002).

Among them, water pricing tools, which are rather commonly used, are also well documented (Johansson, 2000, Roth, 2001, Groom et al., 2006). Their objectives are multiple, well established, and sometimes contradictory: allocating water to users who valorise it at the best, guaranteeing an access to this essential good to everybody, recovering costs induced by water extraction/distribution/use, guaranteeing financial stability to providers, being transparent and simple enough to be understandable, being "acceptable" to be applied, etc. (Tsur et Dinar, 1995, Tsur, 1998).

The economists are interested essentially in the first four ones that correspond to efficiency, equity and cost recovery objectives (Carlson et al., 1993, Zilberman et Lipper, 1997). Here we present an original water pricing device, constructed through 'mechanism design', i.e. by using game theory models in order that the pricing system is able to meet the preceding five objectives (including then intelligibility by water users). We show how introducing some degree of freedom in the system, and using the fact that farmers will keep secret some private information and acquire public information in the course of the plant growth period, may allow the manager of the irrigation area and the farmers to meet these objectives in an acceptable way. Some field data, analysed through econometric tools, confirm this possibility, and the acceptability of such pricing mechanism.

The paper is divided in several sections: firstly we describe the pricing formula, which is a function of water subscription and consumption by each farmer. Secondly, we study the properties of this pricing formula in a deterministic context, when we have either one single farmer, numerous farmers, or finally two farmers. In this last case we determine the Nash equilibrium between them, when they are approximatly similar in their water consumption, or when they are sufficiently different. Then we study the properties of the system, and especially the possible Nash equilibria, in a stochastic environment. The acceptability of such a pricing rule is established elsewhere through an econometric analysis of real data coming from an irrigation area.

2 Modelization

2.1 Notations

We suppose here that a water user association, composed of N farmers, provides them irrigation water coming from a dam of a limited volume. Each farmer, consuming the quantity C_i has in the deterministic case a production function that we note $h_i(C_i)$, a function known only by himself and not by the other farmers nor by the association manager. In the stochastic case some rain π adds to the water consumption, but the function $h_i := h_i(C_i + \pi)$ is unchanged. Each farmer's objective is to maximize the production function less the water bill. The association manager's objective is to present an equilibrated budget, and calling D the total value of the association expenses a given year, the sum of the bills paid by the farmers for the same period must equal this amount.

Each year, an agent firstly reserves a water volume S_i, then consume another volume C_i, either inferior or superior to S_i. The pricing formula is designed in order to fulfill the different objectives or constraints presented before.

The notation we use are the following:

- D is the total water user association expenses,
- N is the number of agents
- S_i is the volume reserved by agent i and S_{-i} the sum of the volumes reserved by the others agents,
- C_i is the volume consumed by agent i and C_{-i} the sum of the volumes consumed by the others agents,
- $S = \sum_{i=1}^{N} S_i$ and $C = \sum_{i=1}^{N} C_i$
- F_i is the sum agent i must pay (the water bill).

Consider C_i given for all i. For each agent i, the pricing formula is:

$$F_i(S_i, S_{-i}) = \frac{1}{2} D \left(\frac{S_i}{S} + \frac{[\max(C_i, 0.7S_i)]^2}{C\ S_i} \right), \quad (1)$$

The pricing scheme is common knowledge for all agents. We will examine some changes of this pricing rule throughout this article.

2.2 Properties of the pricing formula

If C_i is given, the objective of i is to minimize:

$$F_i(S_i, S_{-i}) = \begin{cases} \frac{1}{2} D \left(\frac{S_i}{S_i + S_{-i}} + \frac{(C_i)^2}{C\ S_i} \right) & \text{if } 0.7S_i \leq C_i, \\ \frac{1}{2} D S_i \left(\frac{1}{S_i + S_{-i}} + \frac{0.49}{C} \right) & \text{if } 0.7S_i > C_i. \end{cases}$$

We can deduce the following properties of this pricing scheme:

- $F_i(S_i, S_{-i})$ is a continuous function.
- $$\frac{\partial F_i(S_i, S_{-i})}{\partial S_i} = \begin{cases} \frac{1}{2} D \left(\frac{S_{-i}}{(S_i+S_{-i})^2} - \frac{(C_i)^2}{C\,(S_i)^2} \right) & \text{if } 0.7S_i < C_i, \\ \\ \frac{1}{2} D \left(\frac{S_{-i}}{(S_i+S_{-i})^2} + \frac{0.49}{C} \right) & \text{if } 0.7S_i > C_i. \end{cases}$$

  and it is not defined in $0.7S_i = C_i$.

2.2.1 Case of one single farmer

In the theoretical case of one single farmer (or in other words, in the case where $C_{-i} = S_{-i} = 0$), then

$$F = \begin{cases} \frac{1}{2} D \left(1 + \frac{C_i}{S_i}\right) \text{ if } 0.7S_i \leq C_i \\ \frac{1}{2} D \left(1 + 0.49 \frac{S_i}{C_i}\right) \text{ if } 0.7S_i > C_i \end{cases}$$

We see immediatly that it is possible we have a budget equilibrium, but that it is not a necessity. The bill to pay is at its minimum when $C_i = 0.7S_i$. This is a limit case showing that when the consumptions and subscriptions of others faint to zero, the pricing method does not garantee a budget equilibrium.

2.2.2 Case of numerous farmers

We suppose here that there are numerous farmers, and that the actions of farmer i has no impact on the actions of the other farmers. Moreover we suppose that

$S_i << S_{-i}$, with $S_{-i} = \sum_{j \neq i} S_j$
and $C_i << C_{-i}$with $C_{-i} = \sum_{j \neq i} C_j$

For S_{-i} , C and C_i given,

- if
  $$0.7S_{-i} < \sqrt{S_{-i}\,C} - C_i \tag{2}$$
  then the minimization of (1) is given by:
  $$\frac{\partial F_i(S_i, S_{-i})}{\partial S_i} = 0 \quad \Longleftrightarrow \quad S_i = \frac{S_{-i}\,C_i}{\sqrt{S_{-i}\,C} - C_i}.$$
- if not the minimum of (1) is attained in
  $$0.7S_i = C_i.$$

Notice that in the case where (2) is verified $S_i < C_i/0.7$.

In summary we have always $0.7S_i \le C_i$.

As in this case $F_i(S_i, S_{-i}) = \frac{1}{2}D\left(\frac{S_i}{S} + \frac{C_iC_i}{CS_i}\right)$, if all agents have the same behavior or make the same choice, the budget equilibrium is not garanteed if $0.7S_i \le C_i \le S_i$. In this case, only 85% of the revenues is garanteed. But in fact we may have strategic interactions between farmers. We examine these interactions in the following section, always in the deterministic case, and when there are only two farmers.

2.3 The Nash equilibrium in the deterministic case

2.3.1 Nash equilibrium in the case of two farmers

Now, we are going to compute the Nash equilibrium in consumption C_i and subscriptions S_i, taking into account that consumption is chosen after the subscription decision is done. We consider the simplier case $N = 2$. The problem is:

$$\max_{S_i}\left[\max_{C_i} G_i(S_1, S_2, C_1, C_2)\right], \tag{3}$$

where

$$G_i(S_1, S_2, C_1, C_2) = h_i(C_i) - F_i(S_1, S_2, C_1, C_2),$$

and $h_i(.)$ is an increasing concave fonction of C_i.

2.3.2 The symmetric case

We consider first the case where $h_i = h$ for $i = 1, 2$.
For S_1, S_2 given we compute:

$$\max_{C_i} G_i(S_1, S_2, C_1, C_2). \tag{4}$$

First order condition gives:

$$h'(C_i) = \frac{D}{2}\frac{C_i^2 + 2C_iC_j}{(C_1 + C_2)^2 S_i} \quad \text{if} \quad 0.7S_i < C_i,$$

$$h'(C_i) = -\frac{D}{2}\frac{0.49}{(C_1 + C_2)^2 S_i} \quad \text{if} \quad 0.7S_i > C_i.$$

As h is an increasing function ($h' > 0$) there is no solution of the first order equation when $0.7S_i > C_i$. As h is a concave function (h' is a decreasing function) and $\frac{D}{2}\frac{C_i^2+2C_iC_j}{(C_1+C_2)^2 S_i}$ is an increasing function in C_i there exist a unique solution of the first order condition when $0.7S_i < C_i$. Moreover, as here we consider $h_1 = h_2 = h$ we obtain that $C_1 = C_2 = \bar{C}$. So we can rewrite the first condtion as

$$h'(\bar{C}) = \frac{3D}{8S_i} \tag{5}$$

So, problem (4) has two possible solutions, $C_i(S_i, C_j)$ solution of the first order condition (5) or $C_i = 0.7S_i$. These two posible solutions verify $\frac{\partial C_i}{\partial S_i} > 0$ and$\frac{\partial C_i}{\partial C_j} \geq 0$.
We analyse now, the two differents possibilities (we consider only symmetric solutions because we deal with a symmetric problem):

i) $C_i = 0.7S_i$, $i = 1, 2$. We compute

$$\max_{S_i} \left[h(0.7S_i) - F_i(S_1, S_2, 0.7S_1, 0.7S_2) \right].$$

The first order condition gives: $S_1 = S_2 = \bar{S}$ and $h'(0.7\bar{S}) = \frac{17D}{56\bar{S}}$.

ii) $C_i = C(S_i) > 0.7S_i$ (solution of (5)) , $i = 1, 2$. When solving

$$\max_{S_i} \left[h(C_i(S_i)) - F_i(S_1, S_2, C(S_1), C(S_2)) \right].$$

we obtain that the optimal solution verifies $S_1 = S_2 = \bar{S} = 2\bar{C}$ that is in contradiction with the fact that $C_i = C(S_i) > 0.7S_i$.

We can conclude that in the symmetric case the optimal solution of (3) is for $i = 1, 2$

$$\bar{C} = 0.7\bar{S}, \quad \text{where} \quad h'(0.7\bar{S}) = \frac{17D}{56\bar{S}} \tag{6}$$

2.3.3 The non symmetric case

We suppose here that the two farmers are not the same. They are supposed to have different production functions of the form $h_i(C_i) = \alpha_i ln(1 + C_i)$. Here it is no more possible to derive an analytical solution. So in order to solve this game, we consider that the farmers have two possibilities when choosing their strategies S_i: either $S_i = C_i$ or $S_i = C_i/0.7$, $i = 1, 2$. We also consider $\alpha_1 = 1$, $\alpha_2 = 2$, $D = 1$. When solving the problem (4), for the differents values of S_i, we obtain:

- For $S_i = C_i/0.7$, $i = 1, 2$,

$$S_1^* = 0.3177, \quad S_2^* = 0.1429,$$
$$G_1(S_1^*, S_2^*, C_1^*, C_2^*) = -0.3854, \quad W_2(S_1^*, S_2^*, C_1^*, C_2^*) = -0.0730$$

- For $S_i = C_i$, $i = 1, 2$,

$$S_1^* = 0.2224, \quad S_2^* = 0.1001,$$
$$G_1(S_1^*, S_2^*, C_1^*, C_2^*) = -0.3854, \quad W_2(S_1^*, S_2^*, C_1^*, C_2^*) = -0.0730$$

- For $S_1 = C_1$, $S_2 = C_2/0.7$,

$$S_1^* = 0.2529, \quad S_2^* = 0.1603,$$
$$G_1(S_1^*, S_2^*, C_1^*, C_2^*) = -0.2947, \quad W_2(S_1^*, S_2^*, C_1^*, C_2^*) = -0.0324$$

- For $S_2 = C_2$, $S_1 = C_1/0.7$,

$$S_1^* = 0.2644, \quad S_2^* = 0.0847,$$
$$G_1(S_1^*, S_2^*, C_1^*, C_2^*) = -0.4739, \quad W_2(S_1^*, S_2^*, C_1^*, C_2^*) = -0.0436.$$

It is then easy to verify that the Nash equilibrium for the game in S_i is given by $0.7S_1 = C_1$, $S_2 = C_2$.

2.4 The Nash equilibrium in the stochastic case

We examine here the stochastic case. We suppose that the only stochastic part is the level of rain for the considered geographic area, and that this level is homogeneous for all farmers' fields.

We use here the initial pricing formula (equation 1). We suppose here that there are only two farmers, $i = 1, 2$ and that they interact and place themselves in a Nash equilibrium.

2.4.1 Definition of the risk

In this section, we suppose that the risk at the date of subscrition is only due to the intensity of the rain which we note π, a stochastic value. For the simplest approach, π may be either low ($\pi = \pi_1$) with probability p or high ($\pi = \pi_2 > \pi_1$) with probability $1 - p$. Neither farmer has a better information than his concurrent on this intensity at the date of subscription. Then, at the date of consumption, in the case of a high level of rain, the consumption of water is chosen while S_i is given and π is known.

The production function due to the consumption of the irrigation water C_i is then $h(C_i + \pi)$.

In the following preamble we show that it is possible that the level of rain π_1 and π_2 may be such that when $\pi = \pi_1$, the water consumption C_i is above $0.7S_i$, and when $\pi = \pi_2$, $C_i = 0.7S_i$.

2.4.2 Preliminary result

As in the deterministic case, the farmers find first the Nash equilibrium in S_i as a function of C_i ; they compute afterwards the Nash equilibrium in C_i. The objective of i may be written as:

$$\underset{C_i,S_i}{Max} E\left[h(C_i + \pi) - F(C_i, S_i)\right]$$

a/ If π_2 is sufficiently high, an increase of C_i above $0.7S_i$ will increase the water bill more than it will increase the agricultural revenue. It will be the case if

$$\left.\frac{dh(C_i + \pi_2)}{dC_i}\right|_{C_i=0.7S_i} \leq \left.\frac{dF(C_i, S_i)}{dC_i}\right|_{C_i=0.7S_i}$$

which is equivalent to:

$$\left.\frac{dh(C_i + \pi_2)}{dC_i}\right|_{C_i=0.7S_i} \leq \frac{1}{2}D\frac{0.49S_i^2 + 1.4S_iC_{-i}}{\left(0.7S_i + C_{-i}\right)^2 S_i}$$

However low the value of the right hand term, the left hand term is decreasing in π_2 and can be made lower. This condition is then satisfied, and therefore $C_i \leq 0.7S_i$. As a consumption of at least $0.7S_i$ minimizes the bill when $C_i \leq 0.7S_i$, then $C_i = 0.7S_i$.

b/ If π_1 is sufficiently low, an increase of C_i above $0.7S_i$ will increase the agricultural revenue more than it will increase the agricultural harvest. It will be the case if

$$\left.\frac{dh(C_i+\pi_1)}{dC_i}\right|_{C_i=0.7S_i} > \left.\frac{dF(C_i,S_i)}{dC_i}\right|_{C_i=0.7S_i}$$

which is equivalent to:

$$\left.\frac{dh(C_i+\pi_1)}{dC_i}\right|_{C_i=0.7S_i} > \frac{1}{2}D\frac{0.49S_i^2+1.4S_iC_{-i}}{(0.7S_i+C_{-i})^2\,S_i}.$$

It is difficult to continue without solving the general problem of i, which is done hereafter only. We recall that $C_i \geq 0.7S_i$, since consuming less would decrease the production without decreasing the water bill. The fact that $C_i > 0.7S_i$ is due to the fact that in computing S_i we take into account the possibility of a rainy season. So S_i is less than it would be if we have known that the season would be dry. Therefore the interest to consume more than $0.7S_i$.

2.4.3 The stochastic model

The problem for i is then to choose a level of S_i, so that he maximizes its expectancy of gain at the time of consumption, with this level fixed. It is the same problem for $j \neq i$. We suppose that in the case of drought, the agency is able to provide at least $0.7S_i$ for each farmer i.

We suppose here that the level of rain π_1 and π_2 are such that the preliminary results are verified for agent i. Moreover we take into account here the possibility to take advantage of the information acquisition between the subscription and the consumption and water: We anticipate here the fact that at the time of deciding the consumption, we will know the level of rain. So we do not compute at the time of subscription an optimal level of consumption by maximizing a gain expectancy depending on this last, but we optimize the level of subscription, knowing that at the time of consumption the level of rain will be common knowledge. Notice that the difference between the two optimal values of the agricultural benefits we may compute according these methods is the quasi-option value (Henry, 1974).

We suppose hereafter that the rain level is such that for the two farmers the preliminary results are satisfied. The objective of farmer i is, at the time of subscription:

$$\underset{S_i,C_i}{Max}\left\{\begin{array}{c} p\left[h\left(C_i+\pi_1\right)-\frac{1}{2}D\left(\frac{S_i}{S_i+S_{-i}}+\frac{C_i^2}{(C_i+C_{-i})S_i}\right)\right]+ \\ (1-p)\left[h\left(0.7S_i+\pi_2\right)-\frac{1}{2}D\left(\frac{S_i}{S_i+S_{-i}}+\frac{(0.7S_i)^2}{(C_i+C_{-i})S_i}\right)\right] \end{array}\right\}$$

i.e.:

$$\underset{S_i,C_i}{Max}\left\{\begin{array}{c} -\frac{1}{2}D\frac{S_i}{S_i+S_{-i}}+p\left[h\left(C_i+\pi_1\right)-\frac{1}{2}D\frac{C_i^2}{(C_i+C_{-i})S_i}\right]+ \\ (1-p)\left[h\left(0.7S_i+\pi_2\right)-\frac{1}{2}D\frac{0.49S_i}{(0.7S_i+C_{-i})}\right] \end{array}\right\}$$

We show here that the subscription of water is under that it would have been, having we not taken into account the possibility of a rainy season.

Moreover, we show here that a modification of the anticipation on the rain or drought periods, modelized here by a change in the probability p, leads to a change in the reserved volume of water: an increase in p leads to an increase in S_i.

2.4.4 Example 1: $h(x) = ln(1+x)$ for both farmers (symmetric case)

Consider the case where the farmers have the same production function, called here the symmetric case. This function is supposed to be expressed as $h(x) = ln(1+x)$. As the farmers have an identical profit function, we can anticipate that both are going to consume $C_i = 0.7S_i$ when π_2 is high enough (rainy season). We consider $D = 1$ without lost of generalization. First we compute the Nash equilibrium in C_i, for S_i given when the season is dry. In this case, knowing that $C_i \geq 0.7S_i$ each farmer solves:

$$\max_{C_i} \left\{ ln(C_i + 1 + \pi_1) - \frac{1}{2} \left(\frac{S_i}{S_1 + S_2} + \frac{C_i^2}{(C_1 + C_2)S_i} \right) \right\}.$$

Taking into account the fact that the farmers have the same production function and that in consequence optimal values of S_i are going to be equals, we can conclude that the Nash equilibrium is given by:

$$C_i^* = \frac{8}{3} S_i - 1 - \pi_1$$

We can now compute the Nash equilibrium for S_i. Each farmer must solve:

$$\max_{S_i} \left\{ p \left[ln(\tfrac{8}{3} S_i) - \tfrac{1}{2} \left(\tfrac{S_i}{S_1+S_2} + \tfrac{(\frac{8}{3}S_i - 1 - \pi_1)^2}{(\frac{8}{3}(S_1+S_2) - 2 - 2\pi_1)S_i} \right) \right] + \right.$$
$$\left. (1-p) \left[ln(0.7S_i + 1 + \pi_2) - \tfrac{1}{2} \left(\tfrac{S_i}{S_1+S_2} + \tfrac{0.7S_i}{S_1+S_2} \right) \right] \right\}.$$

Then the symmetric Nash equilibrium $S_i^* = S^*$, $i = 1, 2$, is given by the only positive solution of the equation:

$$(413p - 1323)S^2 + 510(\pi_2 + 1) + p(410\pi_1 - 1810\pi_2 - 1390)S + 600(\pi_1 + \pi_2 + \pi_1\pi_2 + 1) = 0$$

We can verify that the two solutions of this last equation are of different signs because $(413p - 1323)600(\pi_1 + \pi_2 + \pi_1\pi_2 + 1) > 0$.

2.4.5 Example 2: $h(x) = \alpha_i ln(1+x)$ (non symmetric case)

We suppose here that the two farmers are not the same. They are supposed to have different production functions (asymmetric case). Here it is no more possible to derive an analytical solution. So in order to solve this game, we consider that the farmers have two possibilities when choosing their strategies S_i: we consider that either $S_i = C_i$ or $S_i = C_i/0.7$, $i = 1, 2$.

For, $p = 0.5$, $\pi_1 = 0$, $\pi_2 = 1$, $\alpha_1 = 1$, $\alpha_2 = 2$ we find the following Nash equilibrium in C_i when maximazing

$$W_i(S_1, S_2, C_j) := \max_{C_i} \left\{ p \left[\alpha_i ln(C_i + 1 + \pi_1) - \frac{1}{2} \left(\frac{S_i}{S_1+S_2} + \frac{C_i^2}{(C_1+C_2)S_i} \right) \right] + \right.$$

$$\left. (1-p) \left[\alpha_i ln(0.7S_i + 1 + \pi_2) - \frac{1}{2} \left(\frac{S_i}{S_1+S_2} + \frac{0.7S_i}{S_1+S_2} \right) \right] \right\}.$$

- For $S_i = C_i$, $i = 1, 2$,

$$C_1^* = 0.3775, \quad C_2^* = 0.1642,$$

$$W_1(C_1^*, C_2^*, C_2^*) = -0.0758, \quad W_2(C_1^*, C_2^*, C_1^*) = 0.6207$$

- For $S_i = C_i/0.7$, $i = 1, 2$,

$$C_1^* = 0.3013, \quad C_2^* = 0.1342,$$

$$W_1(C_1^*, C_2^*, C_2^*) = -0.0396, \quad W_2(C_1^*, C_2^*, C_1^*) = 0.6221$$

- For $S_1 = C_1$, $S_2 = C_2/0.7$,

$$C_1^* = 0.3863, \quad C_2^* = 0.1368,$$

$$W_1(C_1^*, C_2^*, C_2^*) = -0.0595, \quad W_2(C_1^*, C_2^*, C_1^*) = 0.6150$$

- For $S_2 = C_2$, $S_1 = C_1/0.7$,

$$C_1^* = 0.2917, \quad C_2^* = 0.6202,$$

$$W_1(C_1^*, C_2^*, C_2^*) = -0.0552, \quad W_2(C_1^*, C_2^*, C_1^*) = 0.6202$$

It is then easy to verify that the Nash equilibrium for the game in S_i is given by $S_i = C_i/0.7$, $i = 1, 2$.

Moreover we can analyse the sensitivity of the Nash equilibrium with respect to π_2. We can see that when $\pi_2 = 3$ we obtain

$$W_1(C_1^*, C_2^*, C_2^*) = 0.2676, \qquad W_2(C_1^*, C_2^*, C_1^*) = 1.3200.$$

$$W_1(C_1^*/0.7, C_2^*/0.7, C_2^*) = 0.3048, \quad W_2(C_1^*/0.7, C_2^*/0.7, C_1^*) = 1.320007.$$

$$W_1(C_1^*, C_2^*/0.7, C_2^*) = 0.2948, \qquad W_2(C_1^*, C_2^*/0.7, C_1^*) = 1.31008.$$

$$W_1(C_1^*/0.7, C_2^*, C_2^*) = 0.2795, \qquad W_2(C_1^*/0.7, C_2^*, C_1^*) = 1.3223$$

π_2	C_1	C_2
1	0.3012919717	0.1342213499
3	0.3559692883	0.1867945056
5	0.3865372361	0.1962048460
7	0.4045336441	0.2010852601
9	0.4164148585	0.2040306979
11	0.4248514779	0.2059859585
13	0.4311539831	0.2073717730
15	0.4360420808	0.2084020650
17	0.4399442555	0.2091964164
19	0.4431316626	0.2098266186
21	0.4457843149	0.2103382442
23	0.5608860132	0.2214480304
25	0.5630919585	0.2219673495
27	0.5650016293	0.2224133095

Table 1: $C_i(\pi_2)$, $i = 1, 2$

It is easy to verify that the Nash equilibrium for the game in S_i is given by $S_2 = C_2$, $S_1 = C_1/0.7$.

And when $\pi_2 = 23$, we get:

$$W_1(C_1^*, C_2^*, C_2^*) = 1.156598, \qquad W_2(C_1^*, C_2^*, C_1^*) = 3.12269$$

$$W_1(C_1^*/0.7, C_2^*/0.7, C_2^*) = 1.1946, \quad W_2(C_1^*/0.7, C_2^*/0.7, C_1^*) = 3.12266$$

$$W_1(C_1^*, C_2^*/0.7, C_2^*) = 1.1988, \qquad W_2(C_1^*, C_2^*/0.7, C_1^*) = 3.1092$$

$$W_1(C_1^*/0.7, C_2^*, C_2^*) = 1.156571, \qquad W_2(C_1^*/0.7, C_2^*, C_1^*) = 3.1282$$

It is easy to verify that the Nash equilibrium for the game in S_i is now given by $S_i = C_i$, $i = 1, 2$.

The optimal solutions in C_i as a function of π_2 are given in table 1.

We see here that the value of the objective is increasing with the amount of rain, as expected. But what is more interesting is that the nature of the Nash equilibrium ($S_i = C_i$ or $S_i = C_i/0.7$) changes with the level of rain. When the climate is more rainy (i.e. when the value of π_2 increases, compared to the value of π_1), the farmers are all the more incited to subscribed to a volume equal to their consumption ($S_i = C_i$), and not superior to this value ($S_i = C_i/0.7$). The farmer for which a higher amount (of agricultural goods...) is at stake will be the first to change is subscription volume in this Nash equilibrium, when the level of rain increases.

a	S_1	S_2
0.7	0.4304	0.1917
0.8	0.4093	0.1809
0.9	0.3919	0.1718
1	0.3775	0.1642
1.1	0.3653	0.1575
1.2	0.3549	0.1518
1.3	0.3460	0.1468
1.4	0.3383	0.1423
1.5	0.3316	0.1384

Table 2: $S_i(a)$, $i = 1, 2$

2.4.6 Example 3: $h(x) = \alpha_i ln(1 + x)$ (non symmetric case) whith $C_i = aS_i$ in the dry season

The consumptions are constrained here by the rule that $C_i = aS_i$, when the climate is dry, with $a > 0.7$.

In this situation we only need to compute the Nash equilibrium in S_i. Each agent must solve:

$$\max_{S_i} \left\{ p \left[\alpha_i ln(aS_i + 1 + \pi_1) - \frac{1}{2} \left(\frac{S_i}{S_1+S_2} + \frac{aS_i}{S_1+S_2} \right) \right] + \right.$$

$$\left. (1-p) \left[\alpha_i ln(0.7S_i + 1 + \pi_2) - \frac{1}{2} \left(\frac{S_i}{S_1+S_2} + \frac{0.7S_i}{S_1+S_2} \right) \right] \right\}.$$

For, $p = 0.5$, $\pi_1 = 0$, $\pi_2 = 1$, $a = 0.9$, $\alpha_1 = 1$ we find the following Nash equilibrium:

$$\alpha_2 = 1, \quad S_1 = 0.4865, \quad S_2 = 0.4865$$

$$\alpha_2 = 2, \quad S_1 = 0.3920, \quad S_2 = 0.1719$$

$$\alpha_2 = 3, \quad S_1 = 0.3074, \quad S_2 = 0.0892$$

$$\alpha_2 = 4, \quad S_1 = 0.2519, \quad S_2 = 0.0554$$

Now we consider $p = 0.5$, $\pi_1 = 0$, $\pi_2 = 1$, $\alpha_1 = 1$ and $\alpha_2 = 2$ and we compute optimal solutions for differents values of a, see table 2.

The interpretation is the following: When the part of the water subscribed increases in case of drought, the farmers are incited to subscribe less water when there is uncertainty on the climate.

3 Acceptability

The acceptability of such a pricing system was tested empirically in a French irrigated area. The study of the subscriptions and consumptions of water by the farmers show that firstly they understood well the pricing principles, and secondly that they responded differently according to their culture types. Their response is presented elsewere (Terreaux, 2007) and show that those who would most suffer from a lack of water were reserving a more important water quantity.

4 Comments and conclusion

This pricing system is very interesting since, at the cost of some theoretical analysis, we have shown that it allows the obtainment of some qualities of the water sharing and of the budget equilibrium for the water user association. The study of the properties of such a system is not finished at the present time, but it opens new perspectives in water management, not only in France but for example in Israel, where not only the water quantity is problematic, but the water quality too. Some developments of our model are planed.

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7 Annex

$0 < \frac{0.3}{\sqrt{0.7}}\sqrt{x+x^2} + 1 - 2x^{\frac{3}{2}} = 0.358\,57\sqrt{x}\sqrt{(1.0+x)} + 1.0 - 2.0x^{\frac{3}{2}}$,
Solution is: $\{0 \leq x, x < 0.799\,65\}$