A new method to evaluate weights of decision makers and its application in water resource management

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Abstract:

In a real world group decision making problem, decision makers (DMs) have different knowledge, proficiency and experiences and therefore the importance of each expert against an attribute may not be equal. In many studies this difference of knowledge and experiences (relative weight) of each expert has not been considered and equal weight has been assigned to each expert. Obviously, it is inconsequential and creates imprecise and inaccurate solutions.

This paper presents a new rational approach to evaluate relative weights of decision makers (DMs). In order to assess the relative weights of DMs, this method integrates subjective preferences of group manager and opinions of DMs about themselves simultaneously. The proposed method is general may be applied to great variety of practical problems. A case study (a water resource management problem) is used to illustrate the application of the model to real world decision making processes. Results indicate that rational determination of the DMs' weights should be an integral part of the group decision making process.

Keywords: *Fuzzy Group Decision Making, Decision makers' relative weights, homogeneous and heterogeneous group, Water Resource Management.*

1. Introduction

Socio-economic environment is quite complex, therefore, a single decision maker may fail to consider all relevant aspects of a problem. In such a complex environment, the preference information released by the decision maker may be imprecise or incomplete. Most of the decisions involve the work of a team of experts or specialists and are focused on an analysis and evaluation of attributes in the decision-making process. Consequently, they are, in fact, cases of fuzzy multiple attributive group decision making problems.

In the real world, many decision making processes take place in group settings with incomplete information (Xu and Chen, 2006). In a group decision making problem, decision makers (DMs) have different knowledge, proficiency and experiences. Hence, the group may assign different rate of importance to each expert against an attribute. This difference may be attributed to different ingenuities and abilities of experts, unequal accessibility to available sources and many socioeconomic causes. In many cases, this difference of knowledge and experiences of experts have not been considered and equal weights have been assigned to all experts. Ignorance of relative weights of experts seem to be inconsequential and may create imprecision and inaccuracy in final solutions.

Most often, group manager approximately and subjectively assigns a weight to each DM and based on these weights the best alternative is selected. Weight assignment by the group manager may not be justified

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and may result an irrational solution. Up to now, many methods have been developed to determine the weights of decision makers. Theil (1963) proposed a method based on symmetry when the utilities of the members are measurable. Keeney and Kirkwood (1975), and Keeney (1976) suggested the use of interpersonal comparisons to obtain the values of scaling constants in the weighted additive social choice function. Bodily (1979) derived the member weight as a result of designation of voting weights from a member to a delegation subcommittee made up of other members of the group. Brock (1980) used a Nash bargaining based approach to estimate the weights of group members intrinsically. Ramanathan and Ganesh (1994) proposed a simple and intuitively appealing eigenvector based method to intrinsically determine the weights of group members using their own subjective opinions. Olcer and Odabasi (2005) evaluated the relative importance and weight of each expert was determined comparing each expert with the most important person.

This paper, presents a new method for evaluation of DMs' weights in a group decision making process. It integrates subjective preferences of group manager and assessments of DMs by other members of the team simultaneously. Applications of the methodology have been illustrated in a real world water resources management problem. To do so, the rest of this paper is arranged as follows: Section 2 reviews fuzzy group decision making process. The new method to evaluate relative weights of DMs is presented in Section 3. In section 4, a water resources case study is used to illustrate implementation of the proposed technique. Finally, the last section summarizes this research and concluding remarks are presented.

2. Fuzzy Group Decision Making

In a fuzzy environment, a group decision making problem may be solved in four steps as depicted in Fig.1 (Mianabadi and Afshar, 2006): (1) Specification and Evaluation (2) Unification (3) Aggregation and selection, and (4) consensus measure.

Specification and Evaluation generally includes of (1) identifying DMs; (2) selecting criteria; (3) defining alternatives; (4) eliciting criteria weights; (5) evaluating the importance of each expert, and (6) assessing the performance of alternatives against the criteria. Each expert has its own ideas, attitudes, motivations, and personalities. Different experts give their preferences in different ways; hence, DMs' opinions must be unified employing a unification process. In Aggregation stage, all experts’ opinions are usually combined to form final rating for each alternative. Selection of aggregation function plays an important role in developing the final solution (Chen, 2005). An *aggregation operator* is a function $F : I^n \rightarrow J$ where I and J are real intervals and denote the set of values to be aggregated and the aggregated results, respectively (Smolíková and Wachowiak, 2002). Input vector I and output value J can be linguistic variables, numerical values, or both.

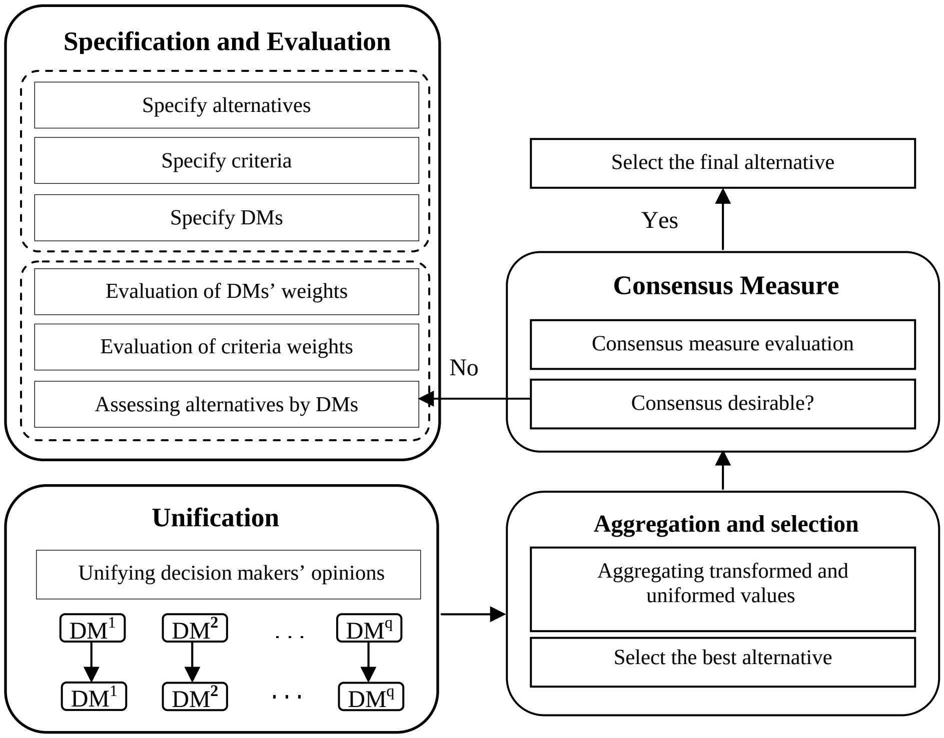

Fig1: *Fuzzy group decision making process (adopted from Mianabadi and Afshar, 2006).*

When a group decision is created and the best possible alternative(s) selected, consensus measure on selected alternative(s) should be evaluated. "Consensus measure" is an index used to evaluate the convergence between group and individuals' opinions. The measure of consensus is calculated by comparing the individual solutions with the global/consensus solution. The group decision makers may set up the consensus level (CL) required for the finally approved solution in advance. When the consensus measure reaches this level, the decision making session is finished and the solutions are obtained (Bryson, 1996).

3. Proposed method to evaluate weights of DMs

The proposed method assumes that each member of the group has background information on the expertise and rationality of other members of the group. The proposed method integrates subjective preferences of the group manager and opinions of the DMs about other members of the team simultaneously to assess the relative weights of DMs.

In the first step, after assessing alternatives by DMs, each decision maker (DM_i) is required to assess relative importance of other experts ($DM_j(j=1,...,t,\ j\neq i)$) as w_{ij}, which defines relative importance of DM_j from DM_i's point of view and $w_{ij}\geq 0,\ \sum_{\substack{j=1\\ j\neq i}}^{n} w_{ij}=1$. The higher the weight, w_{ij}, identifies the greater respect of DM_i for the opinion or expertise of DM_j. In a real world problem, it may not be feasible to ask any individual in the group to assess and assign a weight to other members according to his/her respect on their expertise or views on the issue at hand. Because, it would be a monumental task to obtain individuals' levels of respect on group members in addition to criteria weights and preferences of alternatives and group members may feel distaste to explicitly quantify and reveal the weight of respect for other group members, as it could lead to ill feeling within the group. This is an undesirable outcome when the purpose of the exercise is to reach consensus. Thus, we calculate experts' weights based on the strength of the differences in the opinions expressed by individuals in the group. Higher weights are given by DM^i to these members with similar opinions (or evaluations) and conversely, lower weights are given to members with more diverse opinions.

Assume a group decision making problem where q DMs express their assessments about each alternative on n attribute as follows:

$$\begin{array}{c} \\ DM^1 \\ DM^2 \\ DM^3 \\ \vdots \\ DM^q \end{array} \begin{array}{c} \begin{array}{ccccc} C_1 & C_2 & C_3 & \dots & C_n \end{array} \\ \begin{pmatrix} P_{11} & P_{12} & P_{13} & \dots & P_{1n} \\ P_{21} & P_{22} & P_{23} & \dots & P_{2n} \\ P_{31} & P_{32} & P_{33} & \dots & P_{3n} \\ \dots & \dots & \dots & \dots & \dots \\ P_{q1} & P_{q2} & P_{q3} & \dots & P_{qn} \end{pmatrix} \end{array}$$

Equation 1 is proposed by authors to determine the relative importance of DM_j from DM_i's point of view:

$$w_{ij} = \frac{\sum_{k=1}^{n} \left\{ 1 - \left| P_{ik} - P_{jk} \right|^b \right\}}{\sum_{\substack{j=1 \\ j \neq i}}^{q} \sum_{k=1}^{n} \left\{ 1 - \left| P_{ik} - P_{jk} \right|^b \right\}} \quad , \quad b \in (0,1) \tag{1}$$

The exponent b controls the rigorousness of the weight assigning to each DM, and p_{ik} denotes preference given by DM^i (i=1,…,q) to each alternative on attribute C_k ($k=1,\dots,n$). Obviously, higher weight is assigned to DM closer opinion to DM^i. Overall weight of DM_j according to other members' opinions is calculated by aggregation assigned weights to this decision maker as follows:

$$w_j = OWA(w_{1j}, \dots, w_{j-1,j}, w_{j+1,j}, \dots, w_{qj}) \tag{2}$$

In which ordered weighted averaging (OWA) is an aggregation operator with an associated vector of weights $\sum_{i=1}^{n} w_i = 1, w \in [0,1]^n$ such that (Yager, 1988; 1993; 1994):

$$F_w(x) = \sum_{i=1}^{n} w_i . b_i \quad , \quad x \in I^n \tag{3}$$

with b_i denoting the ith largest element in $x_1;\dots;x_n$. The weighting vector is calculated as follow (Yager, 1993):

$$w_i = Q(\frac{i}{n}) - Q(\frac{i-1}{n}) \quad , \quad i = 1, \dots, n \tag{4}$$

Q is a linguistic quantifier that represents concept of fuzzy majority and is used to calculate the weighting vector. A fuzzy linguistic quantifier may be defined as follows:

$$Q(r)=\begin{cases}0 & \text{if: } r<a \\ \frac{r\text{-}a}{b\text{-}a} & \text{if: } b \le r \le a \\ 1 & \text{if: } r>b\end{cases} \tag{5}$$

where (a, b) are the ranges of linguistic quantifier Q symbolically. The most common linguistic fuzzy quantifiers used are "*most*", "*at least half*", and "*as many as possible*". Their ranges are given as (.3, .8), (0, .5) and (.5, 1), respectively (Choudhurya et al, 2005).

In a real world problem, group manager may have different opinions about relative weights of DMs and be desirous to consider his/her opinion in the decision process. Assuming D_j as the relative importance of DM_j given by the group manager, final weight of each decision maker is calculated as:

$$\lambda_j = \alpha.D_j + \beta.w_j \quad ; \quad \alpha+\beta=1 \tag{7}$$

where α and β are the weight of respect of group manager's opinion and decision makers' opinions in relative weights of DMs elicitation, respectively. When $\alpha = 0$, Eq.7 disregards the group manager opinions and relies considers only on the decision makers' opinions. On the other hand, if $\beta = 0$, then Eq.7 will only consider the group manager's opinion. If α and β are both nonzero, then Eq.7 considers both group manager and DMs' opinions to evaluate relative weights of DMs.

Usually, the manager may want to consider both his/her opinion and DMs' assessments on the weights of members. In this case, a different relative importance can be attached to α and β. One may use the well-known AHP scale 1–9 to determine the values of α and β. Wang and Parkan (2006) proposed for different relations of the values of α and β: if the manager's opinion is thought to be as equally important as the DMs' opinions, then $\alpha = \beta = 1/2$. If the former is thought to be moderately important than the latter, then $\alpha = 3\beta \rightarrow \alpha = 3/4, \beta = 1/4$. If the manager's opinion is considered more strongly important than the DMs' opinions, then $\alpha = 5\beta$, i.e. $\alpha = 5/6, \beta = 1/6$. If the former is regarded as very strongly important than the latter, then $\alpha = 7\beta$ can be set $\alpha = 7/8, \beta = 1/8$. If the former is extremely important than the latter, then the values can be set $\alpha = 9\beta$, i.e. $\alpha = 9/10, \beta = 1/10$. In order to obtain a credible decision results a set of values α and β should be tested more than one to conduct a sensitivity analysis.

4. Case study.

The water resource planning case study is a simplified numerical example of the multi criteria multi participant evaluation model for North China water resources planning (Cai 1994; CIWRHR, 1994). Six water resources development plans are proposed for North China water resources planning. The considered attributes to evaluate these plans are: financial feasibility, economic development, social welfare, environment preservation, agricultural self-sufficiency. The Table 1 shows components of six water resources development plans with respect to five criteria. The projects include local reservoirs, inter basin water divisions, wastewater treatment, and water conservation. The goal of the group is to choose a single plan from the six competing alternatives. For simplicity, Cai et al. (2004) assumed that six decision makers (DM_1-DM_6) have been nominated to participate in the decision process. Each DM gives his or her target numerical value for each indicator with guidance from the decision support system (Table 2). Cai et al (2004) calculated the final score of each plan $x_i(i=1,..,6)$ from the view of each DM as follows:

$$S(i,k) = E(i,j).C(j,k) \tag{8}$$

where $k=1,2,...,K$, $i=1,2,...,I$; and $j=1,2,...,J$, are indices for decision makers, plans and criterion , respectively. E is defined as *single-criterion evaluation matrix*, $e(i, j)$ showing the performance of plan i on criterion j (Table 1); and C, the *individual priority matrix*. Elements of matrix C (i.e. $c(j,k)$) indicates how important criterion j is from the view of DM_k (Table 2); and an element of S, the *support matrix*, $s(i,k)$ indicates the degree of approval that DM_k has for plan i. The final score of each plan from the view of each DM is shown in Table 3.

The ranking system proposed in this study and described in the following is a two stage process. In the first stage, experts' weights with respect to other members' opinions (w_{ij}) are evaluated using the data of Table 3 and Eq.1. In this example, $b=1$ has been considered. Then, the overall weight of each expert with respect to other members' opinions (w_j) is calculated employing Eq.2 (Table 4). We have adopted "*at least half*" quantifier, with the pair (0, .5), and the corresponding OWA operator with the weighting vector w= (0.4, 0.4, 0.2, 0, 0). The evaluated weights should be normalized using the following function:

$$\overline{w}_j = w_j \Big/ \sum_{j=1}^{4} w_j \tag{9}$$

Table 1. *Evaluations matrix*. (Cai et al, 2004).

Plan	Financial feasibility	Economic development	Social welfare	Environment preservation	Agricultural self-sufficiency
I	7	6.5	6.5	3	6
II	3	5	4	5.58	8.5
III	8	6.5	9	0.5	3
IV	5	3.5	4	3.5	6.5
V	1	3	2	6	9
VI	4.5	4.5	4	3	6.5

Table 2. *Priority Matrix, Weights* (Cai et al, 2004).

Criteria	DM^1	DM^2	DM^3	DM^4	DM^5	DM^6
Financial feasibility	0.26	0.13	0.18	0.19	0.20	0.24
Economic development	0.21	0.18	0.24	0.11	0.17	0.26
Social welfare	0.22	0.16	0.24	0.13	0.09	0.18
Environment preservation	0.14	0.28	0.14	0.33	0.29	0.19
Agricultural self-sufficiency	0.17	0.24	0.19	0.24	0.26	0.13

Table 3. *Preference of each alternative with respect to each DM.* (Cai et al, 2004).

Plan	DM^1	DM^2	DM^3	DM^4	DM^5	DM^6
I	6.19	5.74	6.16	5.53	5.62	6.07
II	5.31	5.92	5.55	5.73	5.75	5.31
III	5.99	4.82	5.86	4.50	4.56	5.75
IV	4.95	5.07	5.00	4.93	4.91	4.86
V	3.84	4.81	4.08	4.86	4.87	3.83
VI	4.81	4.85	4.88	4.67	4.73	4.73

Group manager has also presented his /her evaluation about relative importance of each DM. The ratings are chosen from the scale *{1, 2, 3, 4, 5}*. His/her assessment about relative importance of DMs are D_1=1, D_2=4, D_3=2, D_4=4 D_5=3 and D_6=2. The normalized assessments ($\overline{D}_i$) using Eq.9 are shown in Table 4. After negotiations between all members, the manager's and DMs' opinions are considered equally $(\alpha = 0.5, \beta = 0.5)$. The final weight of each DM is calculated as $\lambda =$(0.12, 0.21, 0.15, 0.21, 0.18, 0.15) and presented in Table 4.

Afterwards, in the second stage, the final score of each plan is calculated and the most preferred plan is selected. The final score of each applicant is calculated using OWA operator as follows:

1. The final score of each plan from the view of each DM (Table 3) is multiplied by the relative weight of each expert λ_i (*i=1,2...,6*). In a homogeneous group state, the weight vector of DMs are λ_i =0.16 for *i=1,2,...,6* and in heterogeneous group state, the weight vector of DMs is $\lambda =$(0.12, 0.21, 0.15, 0.21, 0.18, 0.15) as presented in Table 4.
2. Reorder the weighted values in descending order.
3. Multiply these ordered arguments by the OWA weights and then get the final score of each plan. The "*at least half*" quantifier, with the pair (0, .5), and associated weight vector *w*= (*0.33, 0.33, 0.33, 0, 0*) is used to aggregate the experts evaluations to obtain an overall score for each plan. The overall scores of plans are presented for homogeneous and heterogeneous group in Table 5.

For homogeneous group experts, plan 1 is the most preferred alternative whereas, for heterogeneous group of experts, plan 2 is the most preferred alternative (Table 5).

Sensitivity analysis is performed to explore how the values of α and β effect the final values of plans. The Sensitivity analysis is performed for values of α ranging from 0 to 1 as shown in Table 6. Final score of each alternative with respect to different value of α are also given. According to the sensitivity analysis, while $\alpha = 0$ (group manager does not participate in decision process and only DMs evaluate their weights), plan 1 is the best alternative and in the other states, plan 2 is the most preferred alternative.

Table 4. *The overall weights of DMs for* $\alpha, \beta = 0.5$

	w_{ij}						w_i	$\overline{w}_i$	D_i	$\overline{D}_i$	λ_i
	DM[1]	DM[2]	DM[3]	DM[4]	DM[5]	DM[6]					
DM[1]	-	0.20	0.21	0.19	0.20	0.21	0.208	0.169	1	0.06	0.12
DM[2]	0.20	-	0.20	0.20	0.21	0.20	0.204	0.165	4	0.25	0.21
DM[3]	0.21	0.20	-	0.19	0.20	0.20	0.204	0.165	2	0.13	0.15
DM[4]	0.19	0.19	0.21	-	0.21	0.19	0.206	0.167	4	0.25	0.21
DM[5]	0.19	0.21	0.20	0.21	-	0.20	0.208	0.169	3	0.19	0.18
DM[6]	0.21	0.20	0.20	0.19	0.20	-	0.204	0.165	2	0.13	0.15

Table 5. *The final score and rank of each plan for homo/heterogeneous group.*

		Plans					
		I	II	III	IV	V	VI
Score of plan (OWA)	homogeneous	1.02	0.97	0.98	0.83	0.81	0.80
	heterogeneous	1.12	1.15	0.92	0.98	0.96	0.94
Rank of plan	homogeneous	1	3	2	4	5	6
	heterogeneous	2	1	6	3	4	5

Table 6. *Final score of alternative plans with respect to values of* α**.**

α	Plan 1	Plan 2	Plan 3	Plan 4	Plan 5	Plan 6
0	1.02	0.97	0.98	0.83	0.81	0.81
0.1	0.99	1.00	0.94	0.86	0.84	0.82
0.2	1.01	1.04	0.90	0.89	0.87	0.85
0.3	1.04	1.08	0.90	0.92	0.90	0.88
0.4	1.08	1.11	0.91	0.95	0.93	0.91
0.5	1.12	1.15	0.92	0.98	0.96	0.94
0.6	1.15	1.19	0.95	1.02	0.99	0.97
0.7	1.19	1.22	0.97	1.05	1.02	1.00
0.8	1.22	1.26	1.00	1.08	1.05	1.03
0.9	1.26	1.29	1.03	1.11	1.08	1.06
1	1.29	1.33	1.06	1.14	1.11	1.09

The results of alternative ranking in these case studies indicate that consideration and calculation of the DMs' weights should be an integral part of the group decision making process for selection of the most desirable alternative.

5- Conclusion

In a group decision making problem, decision makers (DMs) have different knowledge, proficiency and experiences. Hence, the group might assign different rate of importance to each decision maker against an attribute. In many cases this difference of knowledge and experiences of experts have not been considered and equal weights have been assigned to all experts in the decision making process. Ignorance of relative weights of experts does not seem to be inconsequential and may create imprecision and inaccuracy in final solutions.

This paper, presented a new method for evaluation of DMs' weights in a group decision making process. It integrated subjective preferences of group manager and assessments of DMs by other members of the team simultaneously. In the proposed method, the final alternative was selected in a two general stages. Stage 1 evaluated the DMs' weights and second stage calculated the aggregated and overall value for each alternative. For the first stage, experts' weights with respect to other members' opinions (w_{ij}) were evaluated. Then, applying OWA operator, the overall weight of each expert with respect to other members' opinions (w_j) was calculated.

Application of the method to the real world water resources management problem resulted in different alternative ranking in homogeneous and heterogeneous group. It was concluded that relative weights of decision makers should be taken into account in a more rational decision making process. The results of alternative ranking in this case study indicated that consideration and calculation of the DMs' weights should be an integral part of the group decision making process for selection of the most desirable alternative.

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