

MONTHLY SEASONAL MULTIVARIATE AUTOREGRESSIVE MODEL

Eloy Kaviski

Universidade Federal do Paraná – eloy.dhs@ufpr.br

Miriam Rita Moro Mine

Universidade Federal do Paraná – mrmine.dhs@ufpr.br

Abstract--The objective of the research is to investigate how global climate changes will modify the guaranteed output of a system of interconnected hydropower plants. In particular it is proposed to analyze the performance of the hydropower plants system within the La Plata basin under a set of future climate scenarios. The method should be based on Monte Carlo simulations using synthetic streamflow series representing future scenarios of global climate changes. These series should be obtained from synthetic rainfall series using standard rainfall-runoff models. The rainfall series obtained at monthly time scale need to be generated statistically by defining appropriate stochastic processes to represent them. The generation of synthetic rainfall series is done by the Monthly Seasonal Multivariate Autoregressive Model SMMAR (1) which deals with seasonal by standardizing rainfall and considers non-stationarity in the correlation structure. Time-varying parameters are required to include seasonal variability in the correlation structure. The SMMAR (1) model, applied to the South-Southeast of Brazil, can be satisfactory for the generation of future scenarios of precipitation.

Key-words: rainfall synthetic series, climate changes, hydropower

1. INTRODUÇÃO

The objective of the research project in which this paper is inserted is to investigate how global climate changes will modify the guaranteed output of a system of interconnected hydropower plants. In particular it is proposed to analyze the performance of the hydropower plants system within the La Plata basin under a set of future climate scenarios.

The method should be based on Monte Carlo simulations using synthetic streamflow series representing future scenarios of global climate changes. These series should be obtained from synthetic rainfall series using standard rainfall-runoff models at a monthly time scale. A monthly rainfall-runoff model available is Artificial Neural Networks - ANN for the rainfall-runoff transformation.

The rainfall series obtained at monthly time scale (object of this paper) need to be generated statistically by defining appropriate stochastic processes to represent the rainfall time series. For generation of synthetic series of monthly rainfall the Monthly Seasonal Multivariate Autoregressive Model SMMAR (1) is used.

However, the suitability of these models will depend on the results of a validation study and the availability of an estimate of their parameters under global climate change from the Global Circulation Model - GCM output. These parameters should be provided by GCM and will be needed at the scale of the main sub-basins, because of the important regional diversity of rainfall pattern within the La Plata basin. This part of the study is supposed to be performed by another work group of the Project.

Once the synthetic streamflow series are obtained at the principal sub-basin, disaggregation of these series at individual hydropower plant would be necessary. This can be done by means of standard regionalization techniques used by Brazilian energy sector to transpose streamflows from river gauges to hydropower plant sites.

Next, streamflow series should be converted to natural energy series also using the method used a long time by Brazilian electric utilities in their power supply studies. The natural energy inflows at each hydropower plant are then combined and natural energy hydrograph method will be used to obtain the guaranteed overall energy output for a given energy shortage probability (risk).

For the simulations to be performed it is necessary to have an estimate of future energy demands and how much of these have to be supplied by local plants. Results for several basic climate scenarios should be compared and conclusions about the impact of global climate changes on hydropower generation may be outlined. Models for the natural energy hydrograph method simulation are available both at electric power utilities and at "Universidade Federal do Paraná – UFPR".

Since the CLARIS LPB Project is supposed to be completed within four years this sub-project should be completed at least within the same time. Of course results of GCM models are needed to start the generation of synthetic rainfall series, but the time at this task is performed by others; the suitability of synthetic rainfall series generation models as well as the question of which rainfall-runoff model is best may be analyzed.

It is believed that the proposed study gives a quantitative assessment of the impact of future climate change in generation capacity of all the Brazilian scenarios of the La Plata river basin. This evaluation also allows estimating the evolution of the system's failure risk in the long-term horizon and may guide corrective measures in planning the system's expansion.

It is intended, firstly, present results for three future scenarios (an optimistic, the most likely and a pessimistic), indicating the percentage variance of the guaranteed energy at a 5% level (criterion adopted by "Centrais Elétricas Brasileiras S/A – ELETROBRÁS") in relation to values based on historical series of streamflows. Also should be presented the availability of the firm and secondary energy, calculated from the historical and pseudo-historical series of each scenario. The secondary energy can estimate the viable limit of additional thermal generation. It is intended to show, also, the relation between the guaranteed energy and the risk, which will allow estimating the risk's variance for each of the climate scenarios examined.

Due to uncertainties about the future evolution of the system and the market for electric power, all analysis will be performed for a static configuration constituted by the power plants in the South-Southeast of Brazil. The extrapolation of this study's results to configurations including power plants planned for the future and a development of the market is only viable if the hypothesis of similar impacts of climate change for future systems is reasonable.

This paper presents the development and validation of the Monthly Seasonal Multivariate Autoregressive Model SMMAR (1) that could be used to obtain the series of pseudo-historic rainfall to evaluate the impact of the climate changes in hydropower.

2. CASE STUDY

According to Bajay (2006) Brazil is the largest economy in Latin America and the 9th largest in the world when measured in terms of Purchasing Power Parity exchange rates. It is 10th largest electrical power consumer in the world and the largest electricity consumer in Latin America. Brazil is, therefore, a very important player in the world energy theater.

With a power generation installed capacity of 100,449 MW in 2007, Brazil is the largest electricity market in South America. The generation mix is 76.6% hydro, 21.3% conventional thermal and 2.1% nuclear (MME, 2008).

There are several large hydropower plants, with seasonal or multi-year storage reservoirs. Brazil has a large hydroelectric potential of 260,093 MW, but as of 2007 only 29.6% in actual operation and 2.2% under construction (MME, 2008).

There are approximately 52.2 million customers, 85% of whom are residential users. The 2007 total electricity consumption was 483.4 TWh, distributed among the industrial users (46.7%), residential users (22.1%), commercial users (14.2%) and others (17.0%)."

In Brazil, the system of production, transmission and distribution of electric energy has multiple owners and is interconnected from the east Pará to Rio Grande do Sul, forming the National Interconnected System – SIN (see figure 1).

The SIN consists of the installation of over a hundred agents that includes: generators, transmitters, distributors and free consumers. This organization means that only a small portion (4.2% in 2006) of electrical production capacity of the country is outside of the SIN, in small isolated systems, located mainly in the Amazon region.

The *Operador Nacional do Sistema* - ONS is Brazilian governmental agency responsible for the operation of the SIN. Its performance is targeted towards the search for technical solutions that produce the best outcome for society, respecting the interests of different actors (ONS, 2007).

The South-Southeast interconnected system of Brazil was chosen as a case study of this research, since it is located in *La Plata Basin*. The figure 2 shows a schematic diagram of hydroelectric - SIN - South-Southeast system.

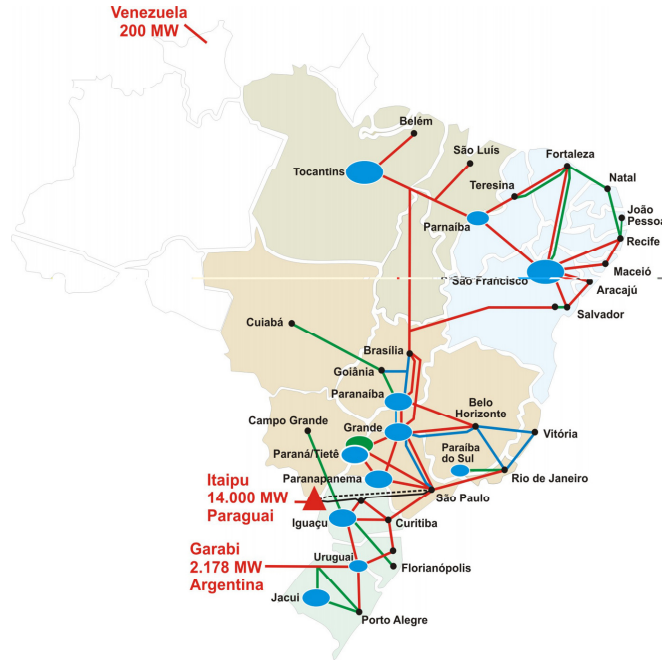
3. THE MODEL THEORY

The theory of the model is presented below. It is used a multivariate autoregressive seasonal model applied to total monthly rainfall transformed by Box-Cox transformation.

This model deals with seasonal by standardizing rainfall and considers non-stationarity in the correlation structure. Time-varying parameters are required to include seasonal variability in the correlation structure. Most widely used is the seasonal AR (1) (Salas and Pegram, 1977; Salas *et al.*, 1980).

The multivariate seasonal autoregressive model of order 1 will preserve all seasonal means of all variables in the state vector, all seasonal variances, all correlations among all elements of the state vector, and lag-one correlations between adjacent seasons and between all variables. As formulated here, the model will be able to handle normal and log-normal variables as well as mixed normal and log-normal or Box-Cox transformation elements in the state vector. The log-normal transformation is a special case of Box-Cox transformation. Mine (1998) used the Box-Cox transformation in rainfall forecasting models.

As the proposed model is valid for stationary processes with marginal normal distribution, the original series of monthly average rainfall should be submitted to a Box-Cox transformation of the first order, with the objective of obtaining data normally distributed.



Source: (ONS, 2007)

Figure 1 - Brazilian integrated system of electricity – SIN

The Multivariate Normal Case

Let

$$(\bar{X}_{i,j} - \bar{m}_j) = \bar{A}_j (\bar{X}_{i,j-1} - \bar{m}_{j-1}) + \bar{B}_j \bar{\epsilon}_{i,j} \quad (1)$$

Where $\bar{X}_{i,j}$ is the state vector ($n \times 1$) of random variables $x_{i,j}^\ell$, during the year i and season j at location ℓ with mean \bar{m}_j .

In detail,

$$\bar{X}_{i,j} = \begin{bmatrix} x_{i,j}^1 \\ x_{i,j}^2 \\ \vdots \\ x_{i,j}^n \end{bmatrix} \quad \text{and} \quad \bar{m}_j = \begin{bmatrix} m_{x_j}^1 \\ m_{x_j}^2 \\ \vdots \\ m_{x_j}^n \end{bmatrix} \quad (2)$$

For example, $x_{i,j}^1$ can be interpreted as the monthly rainfall at station 1 during year i and month j (season) and $m_{x_j}^1$ as the mean value of this variable during the month j .

\bar{A}_j and \bar{B}_j are parameter matrices ($n \times n$), one for each season. The ($n \times 1$) vector of standard normal deviates is $\bar{\epsilon}_{i,j}$, for year i and season j .

Notation for Eq. (1) is simplified by introducing the zero-mean vector, $\bar{Z}_{i,j} = (\bar{X}_{i,j} - \bar{m}_j)$:

$$\bar{Z}_{i,j} = \bar{A}_j \bar{Z}_{i,j-1} + \bar{B}_j \bar{\epsilon}_{i,j}. \quad (3)$$

The similarity between the model given by equation (3) and the multivariate stationary autoregressive lag-one is obvious; the only difference is the seasonal dependence of \bar{A}_j and \bar{B}_j . The lag-zero covariance for season j is ${}_j \bar{M}_0$. The covariance matrix between vectors $\bar{Z}_{i,j}$ and $\bar{Z}_{i,j-1}$ is ${}_j \bar{M}_1$.

Following a derivation analogous (Bras and Rodríguez-Iturbe, 1985) to the stationary models AR (1), it should be easy to arrive at the following results:

$$\begin{aligned}
{}_j \bar{M}_0 &= \bar{S}_{yy} = E[\bar{Y} \bar{Y}^T] \\
{}_j \bar{M}_1 &= \bar{S}_{yx} = E[\bar{Y} \bar{X}^T] \\
{}_{j-1} \bar{M}_0 &= \bar{S}_{xx} = E[\bar{X} \bar{X}^T] \\
\bar{S}_{xy} &= \bar{S}_{yx}^T
\end{aligned} \tag{7}$$

Matrices \bar{S}_{xx} , \bar{S}_{xy} , \bar{S}_{yx} and \bar{S}_{yy} can be represented in terms of variances, standard deviations, and correlations as

$$\bar{S}_{xx} = \begin{bmatrix} S_{x_1}^2 & r_{x_1 x_2} S_{x_1} S_{x_2} & \cdots & r_{x_1 x_n} S_{x_1} S_{x_n} \\ r_{x_2 x_1} S_{x_2} S_{x_1} & S_{x_2}^2 & & r_{x_2 x_n} S_{x_2} S_{x_n} \\ \vdots & & \ddots & \\ r_{x_n x_1} S_{x_n} S_{x_1} & r_{x_n x_2} S_{x_n} S_{x_2} & \cdots & S_{x_n}^2 \end{bmatrix} \tag{8}$$

$$\bar{S}_{yy} = \begin{bmatrix} S_{y_1}^2 & r_{y_1 y_2} S_{y_1} S_{y_2} & \cdots & r_{y_1 y_n} S_{y_1} S_{y_n} \\ \vdots & \ddots & & \vdots \\ r_{y_n y_1} S_{y_n} S_{y_1} & r_{y_n y_2} S_{y_n} S_{y_2} & \cdots & S_{y_n}^2 \end{bmatrix} \tag{9}$$

$$\bar{S}_{yx} = \begin{bmatrix} r_{y_1 x_1} S_{y_1} S_{x_1} & r_{y_2 x_1} S_{y_2} S_{x_1} & \cdots & r_{y_n x_1} S_{y_n} S_{x_1} \\ \vdots & \ddots & & \vdots \\ r_{y_1 x_n} S_{y_1} S_{x_n} & r_{y_2 x_n} S_{y_2} S_{x_n} & \cdots & r_{y_n x_n} S_{y_n} S_{x_n} \end{bmatrix} \tag{10}$$

where S_{x_i} is the standard deviation of variable x_i , $r_{x_i x_j}$ is the lag-zero correlation between stations (variables) x_i and x_j , and $r_{y_i x_j}$ is the lag-one correlation between variables y_i and x_j .

The estimation of sample covariances should again follow the equations (11) and (12) below, to minimize the occurrences of inconsistent (non-positive definite) $\bar{B}_j \bar{B}_j^T$ matrices.

$${}_j \hat{M}_0 = \frac{1}{n} \bar{X}_j \bar{X}_j^T \tag{11}$$

$${}_j \hat{M}_1 = \frac{1}{n} \bar{X}_j \bar{Y}_j^T \tag{12}$$

The Multivariate Log-Normal Case

Consider the case where the elements of vectors \bar{X} and \bar{Y} , x_i and y_i , are random variables following a two-parameter log-normal distribution. Define the variables, x'_i and y'_i , as follows:

$$x'_i = \ln(x_i) \tag{13}$$

$$y'_i = \ln(y_i) \tag{14}$$

Thus, the original variables x_i and y_i are log-normally distributed with means m_{x_i} and m_{y_i} , standard deviations S_{x_i} and S_{y_i} , and the correlation coefficient among them given by $r_{x_i y_i}$.

Therefore, the transformed variables x'_i and y'_i are normally distributed with means $m_{x'_i}$ and $m_{y'_i}$, standard deviations $S_{x'_i}$ and $S_{y'_i}$, and the correlation coefficient among them equal to $r_{x'_i y'_i}$.

To preserve without bias the statistics of the original variables instead of the statistics of the transformed variables it is necessary to compute parameters of the distribution of x'_i and y'_i based on the parameters of the distribution of the original variables x_i and y_i , using the expressions (15) given by Matalas (1967):

$$\begin{aligned}
 m_{x'_i} &= \exp \{ S_{x'_i}^2 / 2 + m_{x'_i} \} \\
 m_{y'_i} &= \exp \{ S_{y'_i}^2 / 2 + m_{y'_i} \} \\
 S_{x'_i}^2 &= \exp \{ 2 [S_{x_i}^2 + m_{x'_i}] \} - \exp [S_{x_i}^2 + 2m_{x'_i}] \\
 S_{y'_i}^2 &= \exp \{ 2 [S_{y_i}^2 + m_{y'_i}] \} - \exp [S_{y_i}^2 + 2m_{y'_i}] \\
 r_{x'_i y'_i} &= \frac{\exp \{ S_{x'_i} S_{y'_i} r_{x'_i y'_i} \} - 1}{\exp \{ S_{x'_i} S_{y'_i} \} - 1}
 \end{aligned} \tag{15}$$

Solving the above system of simultaneous equations for $m_{x'_i}$, $m_{y'_i}$, $S_{x'_i}$, $S_{y'_i}$, and $r_{x'_i y'_i}$ yields

$$m_{x'_i} = \ln (m_{x_i}) - S_{x_i}^2 / 2 \tag{16}$$

$$S_{x'_i}^2 = \ln \left\{ \frac{S_{x_i}^2}{m_{x_i}^2} + 1 \right\} \tag{17}$$

$$m_{y'_i} = \ln (m_{y_i}) - S_{y_i}^2 / 2 \tag{18}$$

$$S_{y'_i}^2 = \ln \left\{ \frac{S_{y_i}^2}{m_{y_i}^2} + 1 \right\} \tag{19}$$

$$r_{x'_i y'_i} = \frac{\ln \{ 1 + r_{x_i y_i} \sqrt{(e^{S_{x_i}^2} - 1) (e^{S_{y_i}^2} - 1)} \}}{S_{x'_i} S_{y'_i}} \tag{20}$$

Thus, having the sample variance and correlation coefficient from the historical records, it is possible to compute the values of the transformed variables using Eqs. (16) through (20).

The parameters of the transformed variables can be used to build the necessary autocovariance and cross-covariance matrices using the definitions given in Eqs. (8), (9), and (10). Generation matrices \bar{A}_j and \bar{B}_j are then available from the previously derived equations.

In order to get the original variables from generated synthetic data, the user must perform the inverse transformation:

$$x_i = \exp (x'_i + m_{x'_i}) \tag{21}$$

$$y_i = \exp (y'_i + m_{y'_i}) \tag{22}$$

Mixture of Normal and Box-Cox Transformation Variables in the Autoregressive Model

Variables such as rainfall are best described by different marginal distributions at each season. For example, the Nile River (Curry and Bras, 1978) exhibits alternating normal and log-normal distributions each month of the year. The multivariate autoregressive model allows the mixing of normal and Box-Cox transformation variables.

Let x_i and y_i be the original random variables. Assume x_i is Box-Cox transformation distributed and y_i is normally distributed as the set of equations (23):

$$\begin{aligned} x_i' &= \ln(x_i) \\ x_i' &\sim N(m_{x_i'}, S_{x_i'}^2) \\ y_i &\sim N(m_{y_i}, S_{y_i}^2) \end{aligned} \quad (23)$$

The procedure to obtain the parameters $m_{x_i'}$ and $S_{x_i'}^2$ the log-normal variables was described in Bras and Rodríguez-Iturbe (1985) and made use of equations. (16) to (20). In this case, the problem is to obtain the correlation coefficient between the log-normal and the normal variables while preserving the parameters of the untransformed data. The necessary expression is given by Mejia *et al.* (1974).

If $r_{x_i y_i}$ is the correlation coefficient among the original variables and $r_{x_i' y_i}$ the correlation coefficient between the transformed log-normal and the normal variables, the following relationship holds,

$$r_{x_i' y_i} = \frac{r_{x_i y_i} \sqrt{e^{S_{x_i'}^2} - 1}}{S_{x_i'}} \quad (24)$$

In order to illustrate how to build the required covariance matrices, consider the following example.

Define the random vectors \bar{X} and \bar{Y} with three random variables each,

$$\bar{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \bar{Y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad (25)$$

where the random variables x_1, y_1, y_2 are normally distributed, and x_2', x_3', y_3' are log-normally distributed. Then the corresponding covariance matrices will be done by equations (26)

$$\begin{aligned} \bar{S}_{xx} &= \begin{bmatrix} (S_{x_1})^2 & r_{x_1 x_2} S_{x_1} S_{x_2} & r_{x_1 x_3} S_{x_1} S_{x_3} \\ r_{x_2 x_1} S_{x_2} S_{x_1} & (S_{x_2})^2 & r_{x_2 x_3} S_{x_2} S_{x_3} \\ r_{x_3 x_1} S_{x_3} S_{x_1} & r_{x_3 x_2} S_{x_3} S_{x_2} & (S_{x_3})^2 \end{bmatrix} \\ \bar{S}_{yy} &= \begin{bmatrix} (S_{y_1})^2 & r_{y_1 y_2} S_{y_1} S_{y_2} & r_{y_1 y_3} S_{y_1} S_{y_3} \\ r_{y_2 y_1} S_{y_2} S_{y_1} & (S_{y_2})^2 & r_{y_2 y_3} S_{y_2} S_{y_3} \\ r_{y_3 y_1} S_{y_3} S_{y_1} & r_{y_3 y_2} S_{y_3} S_{y_2} & (S_{y_3})^2 \end{bmatrix} \\ \bar{S}_{xy} &= \begin{bmatrix} r_{x_1 y_1} S_{x_1} S_{y_1} & r_{x_1 y_2} S_{x_1} S_{y_2} & r_{x_1 y_3} S_{x_1} S_{y_3} \\ r_{x_2 y_1} S_{x_2} S_{y_1} & r_{x_2 y_2} S_{x_2} S_{y_2} & r_{x_2 y_3} S_{x_2} S_{y_3} \\ r_{x_3 y_1} S_{x_3} S_{y_1} & r_{x_3 y_2} S_{x_3} S_{y_2} & r_{x_3 y_3} S_{x_3} S_{y_3} \end{bmatrix} \end{aligned} \quad (26)$$

The above matrices are again routinely used in the existing expressions to obtain \bar{A}_j and \bar{B}_j . The user must remember to add means and perform required inverse transformations at the end of the

computations. Since the elements of the sample covariance matrices are now individually evaluated and result from different nonlinear transformations, it is not unusual to obtain inconsistent estimates for the $\overline{B_j B_j^T}$ matrix.

Box-Cox Transformations

Transformation to normality by taking logarithms is a property of the log-normal distribution; however, in practice, hydrological data follow a probability distribution that is known to be skew but not necessarily log-normal. A more general transformation, of which the log transform is a particular case, is the family of Box-Cox transformations, defined by equations (27)

$$y = \{\exp[\lambda \ln(x)] - 1\} / \lambda, \quad \text{where } \lambda \neq 0$$

$$y = \ln(x), \quad \text{where } \lambda = 0, \quad (27)$$

where λ is to be estimated from the data. The estimation procedure consists of the following:

- i) Assume a range of values for λ : usually values in the range between 2 and -2 are appropriate.
- ii) For each value of λ within the range, transform the y -sequence of data into z -sequence given by the above transformation.
- iii) For each value of λ , calculate the maximum of the log-likelihood function, say $\max[\ln L(\lambda)]$.
- iv) Select the value of λ for which this quantity is maximized.

Clarke (1994) showed that the value of λ which transforms the data sequence from the Rio Hercílio at Ibirama – Brazil is -0.055; clearly this value does not look very different from zero, and we can in fact test whether it differs significantly from that quantity. The procedure to do this is presented by the author in his book.

4. MONTHLY SEASONAL MULTIVARIATE AUTOREGRESSIVE MODEL (SMMAR (1))

To achieve the objective of the paper, the following steps were considered:

- (1) Estimation of model parameters;
- (2) Generation of synthetic time series;
- (3) Analysis of the results.

About to the first of the three items mentioned above we developed a computational method, which was prepared in Pascal Language and implemented in Lazarus. This computational method was divided into three modules that are interconnected through files.

The first module aims at applying the method of moments to estimate the parameters of the Box-Cox transformation. This module prepares a file with the parameters (means, variances and correlation coefficients) for use in the second module.

The second module prepares a file with the synthetic series generated, which can be used by the third module for the analysis of results. The file generated by the second module can be used as an input for other applications to be developed for rainfall-runoff modeling.

The analysis of the results is to verify the conservation properties of the synthetic series compared to historical data used as input data for the first module.

Parameters estimation of the Box-Cox transformation by the method of moments

λ is a parameter that can be estimated by imposing the condition that the skew coefficient of z is zero (corresponding to the skew coefficient $Ca(Z)$ of the normal distribution).

The equation $Ca(Z)=0$ can be solved using the method of simulated annealing (Lee and El-Sharkawi, 2008), estimating λ that minimizes $|Ca(Z)|$. This method is a technique that has recently received much attention for solving large optimization problems (Mckendall et al., 2006). This method was devised by analogy with the Metropolis algorithm (Metropolis et al., 1953), which was proposed to simulate problems of statistical physics by Monte Carlo method (Newman and Barkema, 1999).

The use of the Metropolis algorithm can be described considering a system whose states are not degenerate with energies $E_1 < E_2 < E_3 < \dots < E_n$. For the Metropolis algorithm, the objective of the problem is to simulate the evolution of the system in thermal equilibrium which is at temperature T . It was initially thought that the system is in state j (energy E_j). The system may be carried over to any of the other states f .

The feasibility of the change of state is analyzed by calculating the increase of energy $\Delta E = E_f - E_j$ and the Boltzmann factor $\exp[-\Delta E/(kT)]$, and k is the Boltzmann constant ($1.380658 / 10^{23}$ J/K). We generate a uniform random number p ($0 \leq p \leq 1$) and compare it with the Boltzmann factor: (i) if $p > \exp[-\Delta E/(kT)]$, the proposal is rejected, the system remaining in state j ; (ii) if $p \leq \exp[-\Delta E/(kT)]$, the proposal is accepted, and the system will be changed to the state f .

As a result, if $\Delta E < 0$, i.e. the energy of the system is reduced while transiting, the proposal is always accepted, and if $\Delta E > 0$, the proposal can be accepted with probability p , and if it is accepted, it means that the system energy will increase, and this looks a little different from the idea that a system always tends to decrease its energy, which strengthens and adds importance to the Metropolis algorithm.

To make use of the Metropolis algorithm to solve general problems of optimization, should be known the following (Press et al., 1989): (i) description of the possible system configurations; (ii) a random generator to make configuration changes, these changes are options that should be part of the system settings; (iii) an objective function E (analogy with energy) whose minimization is the goal of the procedure; (iv) a control parameter (analogy with temperature) and an annealing scheme that the system can be conducted to the lowest value of the objective function.

To solve the equation $Ca(Z)=0$ has been developed a computational method where the objective function was defined as $|Ca(Z)|$. The decision variable is the parameter λ . As initial solution is considered $\lambda=0$.

The Metropolis algorithm is used to generate a set of points in a space of distributed variables with probability density function estimated by the Boltzmann factor. The parameter kT initially is defined as 0.5, and subsequently reduced by a factor of 0.9 in each of 100 steps considered. Thus, we will generate a sequence of points (generally identified by the vector λ) representing a random walk moving through space configured.

The rules under which the random walk is performed in space are the following: (i) to consider that the random walk is at the point λ_n ; (ii) to generate the point λ_{n+1} applies an iterative process. The new point can be chosen at random from a range of small δ , around the point λ_n ; (iii) by drafting a possible point λ^* , this solution is accepted or rejected by considering the equation (28):

$$r = \exp[-\Delta E/(kT)] \quad (28)$$

and $\Delta E = E(\lambda_n) - E(\lambda^*)$, where $E(\lambda_n)$ and $E(\lambda^*)$ are the values of the objective function in points λ_n e λ^* , respectively; (iv) If $r > 1$, then the point λ^* is accepted ($\lambda_{n+1} = \lambda^*$), if $r < 1$, the point λ^* is accepted with probability r . This is done by comparing r with a number u uniformly distributed in the interval $[0,1]$, accepting λ^* if $u < r$. When the point λ^* is not accepted the random walk stays in the point λ_n ($\lambda_{n+1} = \lambda_n$) and (v) we generate the point λ_{n+2} using the same procedure.

The value of δ must be chosen so that 1/3 to 1/2 of the configurations generated are accepted, otherwise the method becomes inefficient (Koonin and Meredith, 1993). If there is a lot of rejected configurations it means that the value δ is too large, otherwise if δ is very small, there are many configurations accepted but the region explored by the method is small.

Once determined the parameter λ , the parameters m e s , average and standard deviation of the transformed variable, respectively, can be estimated using the equations (29):

$$\frac{1}{n} \sum_{i=1}^n y_i = \frac{1}{K} \int_0^{\infty} y^\lambda \exp\left[-\frac{1}{2s^2} \left(\frac{y^\lambda - 1}{\lambda} - m\right)^2\right] dy, \quad (29)$$

$$\frac{1}{n} \sum_{i=1}^n y_i^2 = \frac{1}{K} \int_0^{\infty} y^{\lambda+1} \exp\left[-\frac{1}{2s^2} \left(\frac{y^\lambda - 1}{\lambda} - m\right)^2\right] dy,$$

where K is determined by:

$$K = \int_0^{\infty} y^{\lambda-1} \exp\left[-\frac{1}{2s^2} \left(\frac{y^\lambda - 1}{\lambda} - m\right)^2\right] dy. \quad (30)$$

The system of equations (29) must be solved numerically. With the solution obtained the mean and the variance of the variable y (precipitation) are preserved. Analyzing the equations (29) we can conclude that there are solutions in the following cases: $\lambda < -1$ and $\lambda > 1$.

THE SMMAR(1) MODEL

We developed a computational method for generating synthetic series of monthly precipitation. The model used was the multivariate seasonal monthly auto-regressive of order 1 - SMMAR (1), applied to the values of rainfall transformed by Box-Cox transformation.

Implementation of SMMAR(1) model

SMMAR (1) Model was calibrated with the existing monthly rainfall data for the period between Jan/1944 and Dec/2005 in nine stations presented in Table 1 and Figure 3. Depending on the analysis performed were

prepared four versions of the SMMAR (1), which can be used to generate synthetic series of monthly precipitation for these stations.

The proposed versions for the SMMAR (1) model can be classified according to the parameter of the Box-Cox transformation and covariance matrices between series of monthly rainfall (between stations for a given month and the previous month).

SMMAR (1) model can be applied by using the following relationship:

$$y(i)=m(i)+v(i), \quad i=1,\dots,k, \quad (31)$$

where $y(i)$ is Box-Cox transformation of the precipitation in a given month for the site i , in a collection (considered homogeneous) with series of rainfall for n sites; k is the number of sites (series of a given month) with values to be estimated; $n-k$ number of sites with series from the previous month (values considered known); $m(i)$, the average number of $Y(i)$; and $v(i)$ is determined by the expression:

$$v(i)=[z-l(i+1,i)v(i+1)-\dots-l(n,i)v(n)]/l(i,i), \quad i=k,\dots,1, \quad (32)$$

where z is a random number with zero mean and unit variance; $l(i,j)$, with $i=1,\dots,n$ and $j=1,\dots,i$, are elements of a lower triangular matrix obtained by Cholesky factorization of the inverse of the covariance matrix of Y ; the values of $v(i)$, for $i=k+1,\dots,n$, are known, can be initialized or have been determined by the SMMAR(1) model of previous month.

The transformation of Box-Cox variable X , to variable Y , considered normally distributed, is defined by equation (27), where λ is a parameter that can be estimated by imposing the condition that the skew coefficient of y is zero (corresponding to the skew coefficient of the normal distribution).

Table 1A (Appendix A) presents the parameters of the series of monthly precipitation transformed by Box-Cox transformation. The values of λ are estimated by the criterion of skew coefficient zero. The equation $Ca(Y)=0$ was solved using the method of simulated annealing (Lee and El-Sharkawi, 2008), estimated λ that minimizes $|Ca(Y)|$. This Table presents also the mean and standard deviation of Y , represented by $m(Y)$ and $s(Y)$, respectively, and also the results valid for $\lambda=0$ (log transformation). This result is used as the initial solution to the search performed by the method of simulated annealing.

Table 1 Rainfall stations

No*.	Code	Station name	STATE	Lat. (°)(')	Lon. (°)(')
1	01847000	Monte Carmelo	MG	18 45	47 41
2	01848000	Monte Alegre de Minas	MG	18 52	48 52
3	02145007	Usina Couro do Cervo	MG	21 21	45 11
4	02047017	Cidade Nova	SP	20 32	47 25
5	02148128	Fazenda Barreirinho	SP	21 56	48 59
6	02349033	Tomazina	PR	23 46	49 57
7	02651000	União da Vitória	PR	26 14	51 04
8	02851014	Lagoa Vermelha	RS	28 12	51 32
9	02151035	Caiuá (Prefeitura)	SP	21 50	51 59

No* – The serial number of the station identified in each row.

DETERMINATION OF SMMAR (1) PARAMETERS

Analyzing the results shown in Table 1A (Appendix A), it is found that from 108 results, 100 of them are located in the range $0.16 < \lambda < 0.95$ and the rest (8 results) are negative and are in the range $-1 < \lambda < -0.026$.

To estimate the parameters $m(Y)$ and $s(Y)$ of the normal distribution transformed by Box-Cox transformation, by the method of moments should respect the restrictions: $\lambda < -1$ and $\lambda > 1$. All results obtained for λ made it impossible to apply the method of moments, for not comply with these conditions. For this reason it was decided to estimate the parameters $m(Y)$ and $s(Y)$ by applying the method of moments for the normal distribution of rainfall transformed by Box-Cox transformation. The covariance matrices were also estimated in this way. The solution considered, does not preserve the properties of the series of rainfall on the synthetic series generated, but preserves the characteristics of rainfall transformed by Box-Cox transformation.

We decided also to implement the model SMMAR (1) for the log-normal distribution ($\lambda=0$), with all parameters estimated by the method of moments. Applying this model the properties of precipitation series are preserved in the synthetic series generated. In the calibration of this version of SMMAR (1) model, the inverse of the covariance matrices for the months of June, July, August, September and December resulted in matrices that are not positive definite, making impossible the implementation of Cholesky decomposition

(Press *et al.*, 1989). Making small changes in rainfall with values equal to zero, as recommended by Fiering (1968), the matrices are no longer inconsistent and were obtained the Cholesky decomposition.



Figure 3 – Rainfall stations

In summary, the SMMAR (1) model was implemented for each month of the year for precipitation transformed by the logarithm and by Box-Cox transformation. Thus, for each model it is necessary a covariance matrix of 18 dimension for each month of the year (9 rainfall stations with the precipitation of the month and previous month). In the calibration phase of the SMMAR (1) model for the rainfall transformed by Box-Cox transformation, the inverse of the covariance matrices did not show inconsistencies.

The analysis was completed to produce two new models SMMAR (1) for the logarithmic and the Box-Cox transformations, developed on homogeneous groups of stations established through principal component analysis (Mardia *et al.*, 1979). This approach was applied in order to reduce the size of the covariance matrices. For each matrix of correlations between monthly precipitation (one for each month of the year), with 18 dimension, we applied the principal component analysis. So 26 groups were determined, with dimensions ranging between 2 and 12, and the dimensions to 24 groups are smaller than 10.

The stations belonging to each group, according to the month of the year are listed in Table 2. With the exception of month 3, where there is only one group, for the other months there are at least two groups. For each model SMMAR (1) the calibrations were performed with 26 matrices, with dimensions smaller than 12. By reducing the size of the matrices, we avoid the occurrence of inconsistencies in the implementation of the SMMAR (1) model with the log-normal distribution. However, with the reduction in the size of matrices to relax for the preservation of some covariances of monthly precipitation (between stations and / or between a lag one monthly) that by the method of principal components analysis can be considered negligible.

RESULTS OF SMMAR(1) MODEL

A set of 2000 monthly synthetic series with 62 years (historical series size in the observation period between 01/1944 and 12/2005) were generated, for each of the 9 rainfall stations used in this project. SMMAR(1) was applied, which considers the Box-Cox transformation for each month of the year.

By definition, the SMMAR(1) model generates synthetic series of monthly rainfall preserving the characteristics of the historical series. For this reason, to analyze the generated series quality, significance tests were applied to evaluate the characteristics of synthetic annual series.

The estimated parameters are: i) The long term annual average rainfall; ii) the variance of annual rainfall; iii) the 18 dimension correlation matrix, determined between the rainfall annual series (9 rainfall stations with precipitations in the current year and previous years). Tables 1B, 2B and 3B (Appendix B) present for the three parameters the values of 100α (%), where α (%) is the significance test level. Probabilistic distributions valid to each parameter were used to estimate α .

With the tests results, we can verify: i) the long term annual averages rainfall are preserved in all stations with $\alpha > 9\%$ and in eight stations with $\alpha > 16\%$; ii) the variances of annual rainfall are preserved in four stations with $\alpha > 7\%$, in one station with $\alpha > 4\%$, in two stations with $\alpha > 1\%$ and in another two stations with α

<1%; (iii) of the 153 calculated correlation coefficients, 130 are preserved with $\alpha > 10\%$, 14 are preserved with $\alpha > 6\%$, four coefficients with $\alpha > 1\%$ and for only five coefficients, $\alpha < 1\%$.

Table 2 Groups established by analysis of the principal components between of monthly rainfall

Month	Ns	Identification number of the rainfall series (Ni)	Ns	Identification number of the rainfall series (Ni)
1	8	1 2 3 4 5 8 10 14	3	6 7 9
2	7	2 3 8 10 11 15 18	8	1 4 5 6 7 9 12 14
3	12	1 2 3 4 5 6 7 8 9 10 11 12		
4	5	2 7 8 9 10	6	1 3 4 5 6 15
5	7	2 5 6 12 14 15 16	2	9 10
5	4	1 3 4 18	5	7 8 11 13 17
6	6	1 2 3 4 10 12	8	5 6 7 8 9 11 13 14
7	9	1 2 3 4 5 6 7 14 16	3	9 11 18
7	6	8 10 12 13 15 17		
8	8	1 3 5 6 7 8 9 10	10	2 4 11 12 13 14 15 16 17 18
9	8	1 6 7 9 10 11 12 13	6	3 4 14 15 16 18
9	4	2 5 8 17		
10	9	1 2 3 4 5 6 7 8 9		
11	9	1 6 7 8 9 12 13 14 15	6	2 3 4 5 10 11
12	7	2 3 6 7 8 9 10	6	1 4 5 15 17 18

Note Ns – Number of series of monthly precipitation in homogeneous group
 Ni – Identification number of the rainfall series, when
 $Ni \leq 9$ corresponds to the serial number (No) of rainfall station
 from table 1-B ($No = Ni$), when $Ni > 9$, identifies the monthly series in
 the previous month (lag 1) of the station with serial number $No = Ni - 9$.

CONCLUSION

Due the aforementioned results, it is concluded that SMMAR(1) model can be considered as totally satisfactory to produce annual and monthly total rainfall synthetic series, for all the stations considered in this paper.

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Appendix A - Parameters of Box-Cox transformed monthly precipitation series

Table 1A - Parameters of monthly precipitation series transformed by Box-Cox transformation

Code	Month	Log transformation (L=0)			Box-Cox transformation (L≠0)			
		Ca(y)	m(y)	s(y)	L	Ca(y)	m(y)	s(y)
1847000	1	6.3193631	5.359191	1.681039	0.480060	0.0000003	28.654584	9.328324
	2	3.8003748	4.678590	2.583924	0.647140	0.0014031	45.952351	21.539790
	3	4.4852159	4.665891	2.150873	0.577460	0.0004896	31.321211	13.381652
	4	3.7073282	3.634945	1.914335	0.424870	0.0007465	10.662180	5.181721
	5	1.5343964	1.757532	3.221257	0.416030	0.0018576	5.606679	4.745040
	6	0.2262411	-1.053180	3.410711	-0.252120	0.0002761	-3.138835	4.758248
	7	0.2059275	-1.078630	3.531672	-1.000000	-0.0021301	42.023526	42.492477
	8	0.2091887	-1.027562	3.355742	-0.784160	-0.0009373	16.785751	17.663430
	9	1.9423432	2.530566	2.594501	0.418130	0.0010437	7.209995	5.349718
	10	5.7898144	4.489340	1.475407	0.532980	0.0000636	21.405394	7.756070
	11	6.3088619	4.978044	1.564231	0.545550	0.0005524	29.691242	9.971673
	12	6.8107417	5.330294	1.572723	0.731220	0.0000000	78.768537	26.241126
1848000	1	0.1753265	5.592276	0.422796	0.178910	0.0002059	9.655322	1.146388
	2	1.5181980	5.229475	0.560823	0.733070	0.0006379	66.322174	22.709902
	3	0.9772095	5.144749	0.541655	0.734730	0.0003084	62.588886	21.785779
	4	4.5793995	4.019506	1.505941	0.459840	0.0011206	13.420524	5.517202
	5	2.4408504	2.893103	2.001220	0.382260	0.0017718	6.856875	4.244059
	6	0.4756464	0.101904	3.534291	0.211740	0.0011120	1.381262	3.518633
	7	0.1854423	-0.887997	3.501005	-0.266200	0.0001308	-3.125995	4.879091
	8	0.1654474	-0.881519	3.707904	-0.694190	-0.0009675	14.790972	15.880922
	9	1.8569123	2.524440	2.802879	0.688450	0.0001480	15.881888	11.477389
	10	0.8947556	4.793390	0.620151	0.615100	0.0004930	31.474187	11.009120
	11	0.2104654	5.257736	0.415113	0.180490	0.0001326	8.810824	1.068138
	12	0.3959906	5.500518	0.335704	0.437710	0.0002744	23.362368	3.677304
2145007	1	0.7939622	5.465839	0.520554	0.739570	0.0002432	80.992546	27.811256
	2	1.0375728	5.171440	0.625667	0.946290	0.0001807	61.569704	74.005448
	3	0.3786985	5.027965	0.461114	0.224730	0.0005213	9.397490	1.412993
	4	0.7288412	3.976365	0.711963	0.278450	0.0007107	7.481682	2.077438
	5	2.8892567	3.112720	1.805576	0.424670	0.0005142	8.146277	4.622828
	6	0.9282854	0.979754	3.351073	0.359730	0.0012626	3.547621	4.199459
	7	0.5310610	0.271730	3.581951	0.274980	0.0009602	2.054459	3.791725
	8	0.8704639	0.672508	3.336970	0.268670	0.0016651	2.272368	3.519218
	9	2.2550996	2.857084	2.907018	0.448430	0.0003661	9.647205	6.285108
	10	1.6339190	4.584586	0.606638	0.823860	0.0005342	57.340325	22.453600
	11	0.3892138	5.159874	0.520243	-0.070930	0.0024517	4.314408	0.359320
	12	0.3294423	5.534166	0.520458	-0.074320	0.0018218	4.530594	0.343700
2047017	1	0.4704109	5.674215	0.314595	0.508040	0.0004413	33.633394	5.516794
	2	1.5814941	5.361621	0.589433	0.700240	0.0002750	64.047646	21.943620
	3	0.2817746	5.197695	0.426196	0.255990	0.0002089	10.959578	1.600787
	4	1.1533972	4.269883	0.765012	0.532080	0.0007927	17.705471	6.729915
	5	2.8768950	3.322391	1.923707	0.465450	0.0010290	10.403270	5.920510
	6	1.0180254	1.237918	3.348326	0.435490	0.0009819	4.978702	5.074941
	7	0.7227835	0.806016	3.232294	0.273610	0.0009327	2.498432	3.769280
	8	0.6277689	0.555502	3.469357	0.262190	0.0000320	2.263754	3.827412
	9	2.3160155	3.134723	2.656936	0.475940	0.0016308	11.560110	7.416688

10	0.0803321	5.013592	0.542240	-0.026000	0.0014848	4.697209	0.475823
11	0.0991230	5.330110	0.372082	0.162210	0.0001722	8.497553	0.882284
12	1.1476025	5.582724	0.388864	0.689220	0.0006235	68.839955	16.874255

2148128	1	1.2204017	5.318549	0.569724	0.750700	0.0006482	76.716036	27.547319
	2	0.3664328	5.191018	0.521170	0.296100	0.0000179	12.514535	2.392228
	3	0.6779136	4.747619	0.600707	0.221440	0.0004402	8.517725	1.678648
	4	2.9750010	3.467787	1.999019	0.525980	0.0005282	13.548511	7.666010
	5	2.7725588	3.252533	2.339366	0.434320	0.0014061	9.940159	5.788369
	6	1.8030332	2.360990	2.906244	0.454250	0.0013332	7.843618	5.964460
	7	1.1258860	1.308811	3.253342	0.257300	0.0013771	3.124318	3.850079
	8	0.7002651	0.776492	3.673323	0.319370	0.0003672	3.339571	4.513460
	9	2.9101109	3.521883	1.624139	0.538180	0.0006193	13.520529	7.517169
	10	0.6827180	4.622825	0.508081	0.458690	0.0000018	16.465347	4.071380
	11	2.9910594	4.644360	0.779305	0.465930	0.0008633	17.499023	5.362348
	12	0.3609473	5.238983	0.442586	0.249120	0.0006214	10.879997	1.615972

2349033	1	0.8687163	5.229799	0.499188	0.404890	0.0007280	18.456369	3.970501
	2	1.0651076	4.938956	0.636842	0.613040	0.0002098	34.383667	11.974723
	3	0.4312758	4.817958	0.510165	0.335640	0.0003464	12.248085	2.525437
	4	4.0805393	4.021977	1.560331	0.554320	0.0013323	18.308334	8.541584
	5	4.1721175	4.077790	1.563754	0.361090	0.0000278	10.499667	4.461986
	6	3.5942491	3.911189	1.603288	0.532660	0.0007556	16.396195	8.230452
	7	2.6070115	3.195320	2.628986	0.429020	0.0016806	10.105902	6.072232
	8	2.2424340	2.998327	2.405628	0.499150	0.0007184	10.919889	7.295228
	9	0.7933689	4.327763	0.845106	0.410500	0.0006757	12.772412	4.705025
	10	0.5357068	4.827226	0.542653	0.357390	0.0005416	13.197423	2.973251
	11	5.7983097	4.427254	1.422779	0.637880	0.0014011	29.224667	11.056759
	12	0.5312498	4.997251	0.464191	0.422200	0.0003848	17.530345	3.736735

2651000	1	0.7516299	5.012819	0.486906	0.593930	0.0003467	32.686183	9.090260
	2	1.3813989	5.009812	0.544311	0.756420	0.0006896	61.495844	21.193453
	3	0.4015388	4.812256	0.509299	0.357390	0.0003818	13.080082	2.798225
	4	1.4078514	4.511486	0.833150	0.259190	0.0025304	8.832413	2.472924
	5	0.6139751	4.598691	0.877723	0.340970	0.0002747	11.739578	4.041232
	6	1.4879697	4.712455	0.761504	0.753620	0.0003390	51.260124	22.466030
	7	1.1025503	4.577532	0.823443	0.283470	0.0008226	9.715070	2.818391
	8	0.9700424	4.435699	0.871449	0.397400	0.0001174	12.961817	4.698800
	9	0.5924168	4.931120	0.630008	0.427740	0.0002286	17.604600	5.014737
	10	0.2073749	5.146172	0.454904	0.235520	0.0004370	10.103211	1.521235
	11	1.2768009	4.813408	0.619290	0.600290	0.0001640	30.142880	9.996949
	12	0.4111784	4.987509	0.509669	0.267990	0.0007472	10.601421	1.913171

2851014	1	0.3258201	5.089203	0.558614	0.263470	0.0003940	10.867370	2.112445
	2	0.8576770	4.958595	0.605431	0.695910	0.0000580	47.565998	17.712930
	3	0.7285735	4.727738	0.601162	0.383010	0.0007957	13.761262	3.529627
	4	2.5125156	4.553465	1.048034	0.582740	0.0010005	25.969541	11.261698
	5	1.7485729	4.657253	0.972333	0.570630	0.0011224	26.332364	11.272929
	6	4.0265446	4.773395	0.981558	0.596880	0.0010862	30.381683	11.335228
	7	2.0565471	4.837057	0.785248	0.264330	0.0004039	10.066809	2.519741
	8	2.3310016	4.765468	1.078548	0.582770	0.0002468	29.863226	13.086968
	9	0.6688609	5.184278	0.527101	0.471120	0.0002019	23.012052	5.826530
	10	0.3162138	5.224769	0.493854	0.255800	0.0003613	11.085744	1.861556

11	1.2218026	4.772438	0.669702	0.454490	0.0009718	17.866900	5.364550
12	2.3103425	4.905425	0.715652	0.525670	0.0008263	24.613838	7.705147

2151035	1	0.6990493	5.148065	0.668286	0.320110	0.0002626	13.467090	3.354696
	2	1.4010966	4.850543	0.790256	0.461730	0.0001886	19.367535	6.566269
	3	5.6605243	4.447254	1.439823	0.614820	0.0005643	27.574916	10.901806
	4	3.5000138	3.719105	2.016132	0.478260	0.0005328	13.303194	6.678317
	5	3.4216159	3.754710	2.052103	0.443510	0.0003388	12.371004	6.384190
	6	1.9556380	2.580352	2.989109	0.373940	0.0003638	7.183628	5.368737
	7	1.2378174	1.622520	3.450602	0.311830	0.0011648	4.484576	4.625186
	8	0.9123537	1.151510	3.586246	0.301560	0.0012410	3.683434	4.515031
	9	3.1725336	3.539110	1.944404	0.457850	0.0008233	11.420409	6.265944
	10	0.7821386	4.603464	0.787281	0.348330	0.0009268	11.908756	3.728820
	11	5.0419424	4.398756	1.653284	0.334660	0.0000005	11.141970	4.119752
	12	0.6386761	4.941557	0.696518	0.283520	0.0010627	11.061150	2.738792

Appendix B – Test results

Table 1B - Long term annual average

Station	Annual Average	100- α (%)	Station	Annual Average	100- α (%)
1	1461.57	76.58	2	1515.32	90.77
3	1483.18	79.34	4	1710.91	78.65
5	1307.36	77.17	6	1428.24	70.33
7	1780.93	65.65	8	1957.72	67.91
9	1290.05	83.56			

Table 2B - Annual variance

Station	Annual Variance	100- α (%)	Station	Annual Variance	100- α (%)
1	169419.9	92.9	2	54853.93	95.64
3	258161.4	99.9	4	92036.31	99.39
5	58842.7	91.47	6	53846.15	69.84
7	120867.3	99.8	8	193348.4	99.95
9	82778.9	55.36			

Table 3B - Correlation coefficients

Stations	Correlation Coef	100- α (%)	Stations	Correlation Coef	100- α (%)		
2	1	0.099192	20.9	3	1	0.222375	46.97
3	2	0.428221	58.45	4	1	0.261139	83.46
4	2	0.51431	78.08	4	3	0.35497	3.31
5	1	0.270424	89.53	5	2	0.211369	62.17
5	3	0.190441	48.29	5	4	0.239694	39.66
6	1	0.228727	92.34	6	2	0.164898	69.17
6	3	0.14496	54.4	6	4	0.185292	46.51
6	5	0.444469	59.47	7	1	-0.0211	53.94
7	2	0.082175	75.81	7	3	0.076762	66.65
7	4	0.244876	88.32	7	5	0.233935	58.45
7	6	0.578717	84.45	8	1	0.027888	80.61
8	2	0.040055	86.38	8	3	0.022761	69.29
8	4	0.177137	91.52	8	5	0.158116	61.42
8	6	0.376658	75.39	8	7	0.616476	77.03
9	1	0.07957	78.39	9	2	0.253193	83.02
9	3	0.025509	38.75	9	4	0.283776	62.44
9	5	0.23555	21.04	9	6	0.336053	18.6
9	7	0.42026	84.06	9	8	0.230676	72.16
10	1	-0.13726	22.21	10	2	0.094954	78.47
10	3	-0.03738	41.77	10	4	0.077332	76.97
10	5	-0.02653	37.3	10	6	-0.01263	49.13
10	7	0.226694	95.47	10	8	0.335764	99.41
10	9	0.247001	91.34	11	1	-0.09268	29.69
11	2	-0.13075	20.09	11	3	-0.13014	23.33
11	4	-0.01321	49.5	11	5	-0.21938	4.59
11	6	0.118464	77.46	11	7	0.17382	89.88
11	8	0.180617	89.2	11	9	0.142097	82.29
11	10	0.107754	22.9	12	1	-0.1367	29.24
12	2	-0.04896	44.61	12	3	-0.08968	30.13

12	4	0.050471	55.93	12	5	-0.25484	0.62
12	6	0.009083	35.73	12	7	0.119851	77.23
12	8	0.319599	97.77	12	9	0.042589	51.87
12	10	0.436736	95.48	12	11	0.483219	77.64
13	1	-0.14816	16.04	13	2	-0.00857	45.34
13	3	-0.10937	13.77	13	4	0.075094	62.84
13	5	-0.17548	4.62	13	6	-0.01446	28.24
13	7	-0.07951	20.75	13	8	0.204162	92.95
13	9	-0.0565	17.56	13	10	0.265508	84.87
13	11	0.520647	80.06	13	12	0.630252	85.93
14	1	-0.03592	41.6	14	2	-0.01386	55.64
14	3	0.157488	89.98	14	4	0.100028	75.57
14	5	-0.09303	25.07	14	6	-0.0027	57.56
14	7	0.178081	92.45	14	8	0.336955	99.79
14	9	-0.1647	11.96	14	10	0.27634	90.67
14	11	0.207724	60.75	14	12	0.29858	80.62
14	13	0.252489	43.64	15	1	-0.19035	8.77
15	2	-0.019	51.38	15	3	-0.2094	8.5
15	4	0.061501	71.26	15	5	-0.02185	50.96
15	6	0.162075	92.8	15	7	0.296692	99.29
15	8	0.36431	99.83	15	9	-0.04855	47.79
15	10	0.234998	93.24	15	11	0.158892	67.39
15	12	0.201405	69.9	15	13	0.196917	50.08
15	14	0.451187	61.44	16	1	-0.30271	0.78
16	2	-0.01076	42.88	16	3	-0.20884	6.6
16	4	-0.0443	43.4	16	5	-0.02461	50.23
16	6	-0.0666	37.25	16	7	0.113281	85.86
16	8	0.154688	91.09	16	9	-0.21525	6.02
16	10	-0.0139	56.02	16	11	0.08967	77.78
16	12	0.204622	92.15	16	13	0.259174	90
16	14	0.250027	64.66	16	15	0.587481	87.28
17	1	-0.19752	8.44	17	2	0.020009	61.74
17	3	-0.26577	3.08	17	4	0.02694	65.32
17	5	0.019867	60.22	17	6	-0.17252	11.26
17	7	-0.04442	38.61	17	8	0.093853	82.08
17	9	-0.12389	16.67	17	10	0.038217	82.38
17	11	0.036319	84.59	17	12	0.038582	73.74
17	13	0.19611	93.54	17	14	0.176737	67.8
17	15	0.387084	79.1	17	16	0.628003	81.3
18	1	-0.1385	18.22	18	2	0.009958	47.08
18	3	-0.00099	52.16	18	4	0.118118	79.11
18	5	0.152307	91.48	18	6	0.216403	98.59
18	7	0.31887	99.73	18	8	0.194925	93.41
18	9	0.075722	77.58	18	10	0.093963	82.04
18	11	0.239258	80.32	18	12	0.044326	44.36
18	13	0.307509	70.24	18	14	0.2606	28.13
18	15	0.349667	22.24	18	16	0.4435	88.29
18	17	0.270674	82.17				
