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# *Developing a Non-stationary Drought Index Using a Generalized Additive Model for Drought Risk Assessment*

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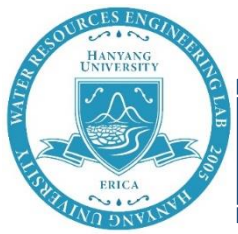
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# Introduction

## Problem in Practice

- Drought is a complex natural phenomenon that usually begins with the deficit of precipitation, leading to agricultural, hydrological and socio-economic drought (Li et al. 2013).
- The detection of onset and the end of a drought event is difficult to access.
- Drought events, also called “the creeping disaster”, develop slowly and often unnoticed and have diverse and indirect consequences (Wilhite 2000; Mishra and Singh 2010).
- Global warming is currently a crucial issue, which increases the intensity and frequency of drought events and also has direct impacts on the availability of water resources (Kabat et al. 2003; Leng et al. 2015).
- Droughts are strictly related to stochastic phenomena, so probabilistic characterization and knowledge about the frequency of drought are needed for effective drought management strategies (Mishra and Singh 2011).



# Introduction

## Drought Impacts

- About half of the earth's terrestrial surfaces are susceptible to droughts and more importantly, almost all the major agricultural lands are located there (USDA, 1994).
- In terms of damages such as crop yield reduction, economic costs and the number of people effected, the drought are on top among other natural disasters (Obasi, 1994; Wilhite, 2000; Hewitt, 2014).
- The estimated annual cost of damages caused by drought varied from 6 to 8 billion USD\$ which is higher than any other natural disaster (Hao et al., 2014; Mao et al., 2015).



Corn crop damaged by severe drought in Colorado and Bruceville, Indiana in 2012.

# Introduction

## Drought in South Korea

- United Nations classified South Korea as a country suffering from more than moderate water shortage and experiencing serious droughts since 1990s (Choi et al. 2008), and drought risk is likely to increase over the course of the twenty-first century due to climate change (Boo et al. 2004; Yoo et al. 2012).
- South Korea experienced large scale droughts in every two years (Kim et al. 2011).
- Due to the highly concentrated rainfall pattern in summer, the vulnerability of drought increases in other seasons (Yoo et al. 2015).

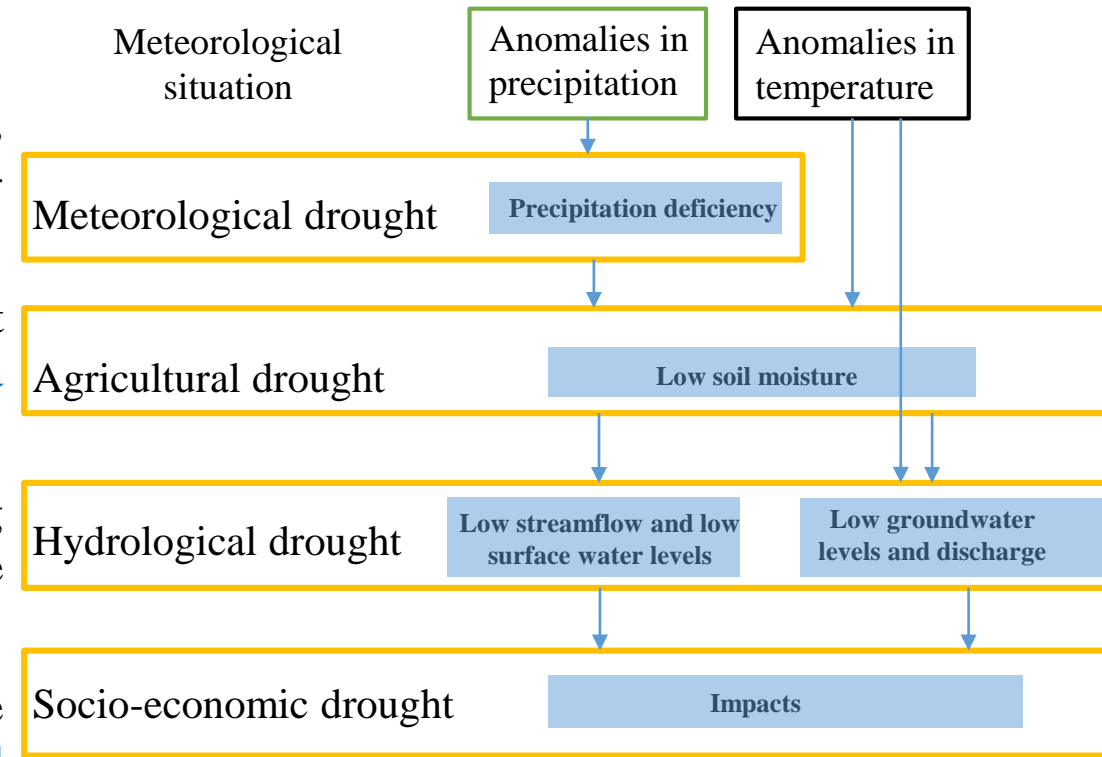


The dried out Soyang lake and sea bed of the Soyang River in Chuncheon, Gangwon Province, northeastern South Korea 2015.

# Introduction

## Drought Types, Drought Indices and Problems

- Droughts are generally classified into four categories; meteorological, agricultural, hydrological and socio-economic drought (Wilhite and Glantz, 1985).
- SPI, SSI, & SRI have been widely used to detect meteorological, agricultural and hydrological drought (Mishra and Singh 2010).
- Standardized drought indices were calculated by fitting probability distribution to hydro-meteorological variable based on the assumption of stationarity (Wang et al. 2015)
- These approaches for calculating drought indices are true when the climate is stable or stationary (Bazrafshan and Hejabi 2018).
- Under climate instability or non-stationarity circumstances, drought analysis using stationary drought indices may lose their generality and validity (Ling and Jiang 2018).

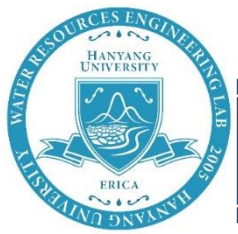


Scheme representing different types of drought and their development

# Introduction

## Objectives of Study

- Frequency analysis is a commonly used method in hydrologic modeling.
- The copula method has proven to be a robust tool in multivariate cases and has been applied in non-stationary research in recent years.
- The bivariate evaluation of different drought indices under non-stationary assumption has not done so far.
- In this study, we proposed a non-stationary Standardized Precipitation Index ( $SPI_{NS}$ ) that is sufficiently robust to monitor drought under non-stationary conditions.
- Finally, this study evaluate the effect of non-stationary properties of meteorological drought on bivariate frequency analysis and compare it with the conventional practices.

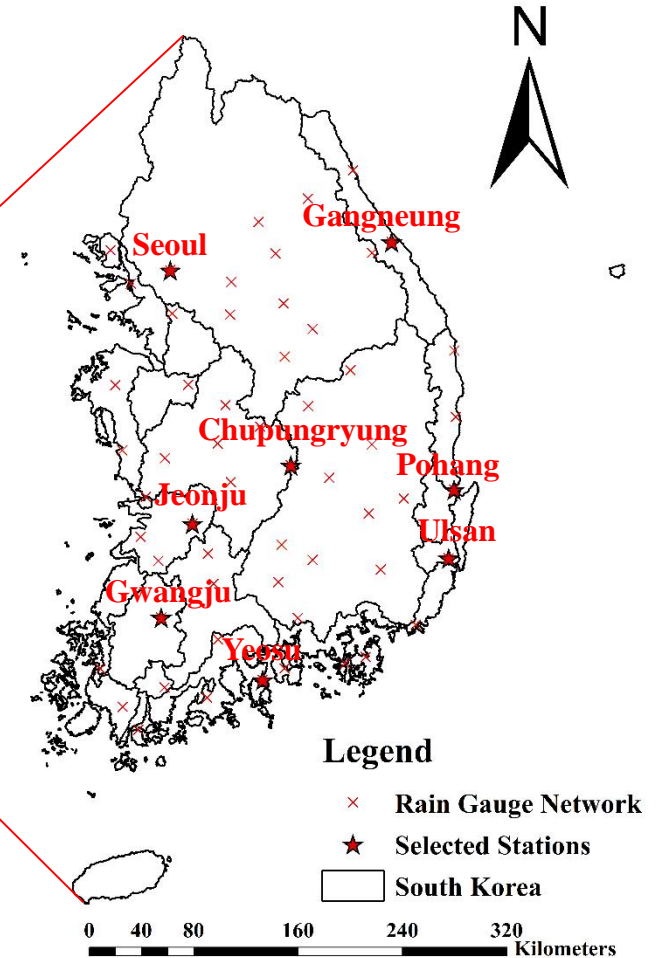
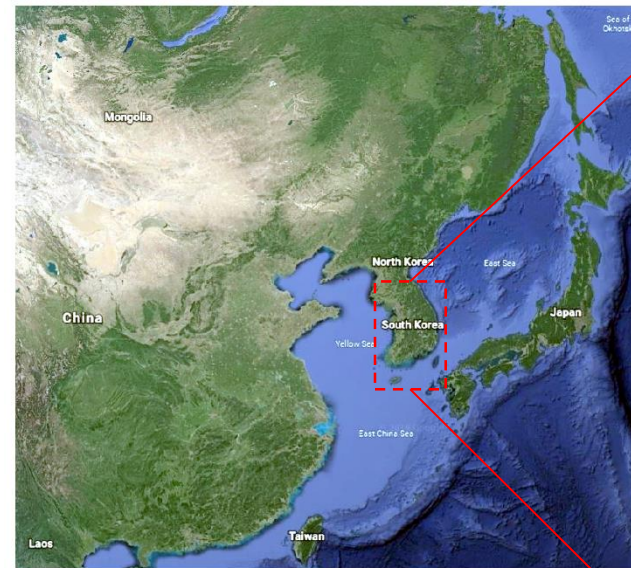




# Description of Study Area

## Research Area and Data

- ❑ The study area is located in South Korea at  $34^{\circ}$  -  $38^{\circ}$  latitude and  $126^{\circ}$  -  $130^{\circ}$  longitude.
- ❑ Out of 56 rain gauge stations 8 rain gauge stations were selected.
- ❑ The mean annual precipitation ranges between 800-1800mm. More than 50% precipitation falls in summer season.
- ❑ The average annual temperature ranges between  $10^{\circ}\text{C}$  -  $14.5^{\circ}\text{C}$ .
- ❑ Precipitation data period 1961-2013.
- ❑ <http://www.kma.go.kr/>



# Estimation of Drought

## Conventional Method

- Standardized Precipitation Index (SPI) was used to calculate meteorological drought at 12 month time scale.
- In order to calculate SPI, firstly the daily data is aggregated into monthly time scale and appropriate marginal distributions (gamma) is fitted to long term Precipitation series.
- Many studies (McKee et al. 1993; Wang et al. 2015) found in literature describe that log-normal distribution is well fitted to streamflow data.
- The cumulative distribution function (CDF) of fitted marginal distribution is then transformed into standard normal variate with zero mean and one standard deviation.

$$SRI = \begin{cases} -\left(t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3}\right), & t = \sqrt{\ln\left(\frac{1}{F(x)^2}\right)}, 0 < F(x) \leq 0.5 \\ \left(t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3}\right), & t = \sqrt{\ln\left(\frac{1}{1 - F(x)^2}\right)}, 0.5 < F(x) \leq 1 \end{cases} \quad (1)$$

where,  $F(x)$  is the cumulative probability distribution function, and  $c_0, c_1, c_2, d_1, d_2,$  and  $d_3$  are constants.

## Non-stationary Method

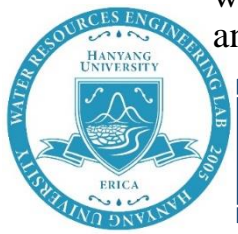
- In this study, Generalized Additive Models for Location, Scale and Shape (GAMLSS), which was proposed by (Rigby and Stasinopoulos 2005) was used to model precipitation series with non-stationary probability distributions.
- In a GAMLSS framework, for observations of a variable  $y$  that follows a distribution with PDF  $f(y|\theta)$ , the parameter vector  $\theta^T$  is described as a function of explanatory variables and random effects.
- In case there are no additive terms, the distribution parameters can be denoted by a monotonic link function  $\eta_k$  as

$$\eta_k(\theta_k) = X_k \alpha_k = \alpha_{0k} + \alpha_{1k} X + \dots + \alpha_{qk} X^q, \quad k = 1, 2, \dots, p \quad (2)$$

where

$X_k$  denotes the explanatory variables;  
 $\alpha_k$  are polynomial coefficients;  
 $p$  = number of parameters;  
 $q$  = degree of polynomial;

- The location and scale parameters were related to explanatory variables in this study, including that one of them is time varying or that both of them are time varying. The CDF of non-stationary probability distribution is transformed into standard normal variate using Eq. (1).



# Frequency Analysis

## Univariate Return Period

- The univariate return periods are expressed by following equations:

$$T_S = \frac{E(L)}{1-F_S(s)} \quad (3)$$

$$T_D = \frac{E(L)}{1-F_D(d)} \quad (4)$$

where  $T_S$  and  $T_D$  represent the univariate return period of drought severity ( $S$ ) and duration ( $D$ ) to be greater than or equal to a given drought severity ( $s$ ) and duration ( $d$ );  $F_S$  and  $F_D$  are the cumulative distributions for severity and duration.

## Conditional Drought Distribution

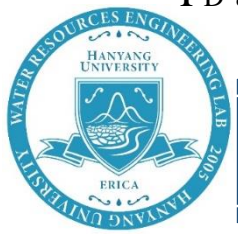
Conditional drought distribution were also derived from copula based bivariate distribution using following equations.

- To evaluate the drought severity distribution given that the drought duration  $D$  exceeding a certain threshold  $d$ .

$$P(S \leq s | D \geq d') = \frac{P(D \geq d', S \leq s)}{P(D \geq d')} \quad (5)$$

- To evaluate the drought duration distribution given that the drought severity  $S$  exceeding a certain threshold  $s$ .

$$P(D \leq d | S \geq s') = \frac{P(D \geq d, S \leq s')}{P(S \geq s')} \quad (6)$$



# Frequency Analysis

## Joint Return Period

- The joint return periods of drought events are estimated using two conditions:
  - The return period  $T_{DS}$  for  $D \geq d$  and  $S \geq s$  when both drought duration and severity exceed their specified values.

$$T_{DS} = \frac{E(L)}{P(D \geq d \text{ and } S \geq s)} = \frac{E(L)}{1 - F_D(d) - F_S(s) + F_{D,S}(d,s)} \quad (7)$$

- the return period  $T'_{DS}$  for  $D \geq d$  or  $S \geq s$  when either drought duration or severity exceed their specified values.

$$T'_{DS} = \frac{E(L)}{P(D \geq d \text{ or } S \geq s)} = \frac{E(L)}{1 - F_{D,S}(d,s)} \quad (8)$$

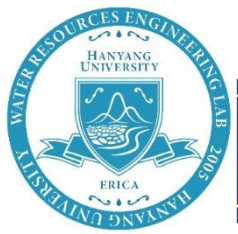
## Conditional Return Period

- The conditional return periods of drought events are estimated under two conditions:
  - The return period of drought duration  $D$  given drought severity exceeding a threshold  $s$ .

$$T_{D|S \geq s} = \frac{T_S}{P(D \geq d \text{ and } S \geq s)} \quad (9)$$

- The return period of drought severity  $S$  given drought duration  $D$  exceeding a threshold  $d$ .

$$T_{S|D \geq d} = \frac{T_D}{P(D \geq d \text{ and } S \geq s)} \quad (10)$$



# Results

## Comparison between SPI<sub>C</sub> & SPI<sub>NS</sub>

Seoul

Gwangju

Non-Stationary SPI CDF

Conventional SPI CDF

Non-Stationary SPI CDF

Conventional SPI CDF

Pohang

Non-Stationary SPI CDF

Conventional SPI CDF

Ulsan

Non-Stationary SPI CDF

Conventional SPI CDF

Jeonju

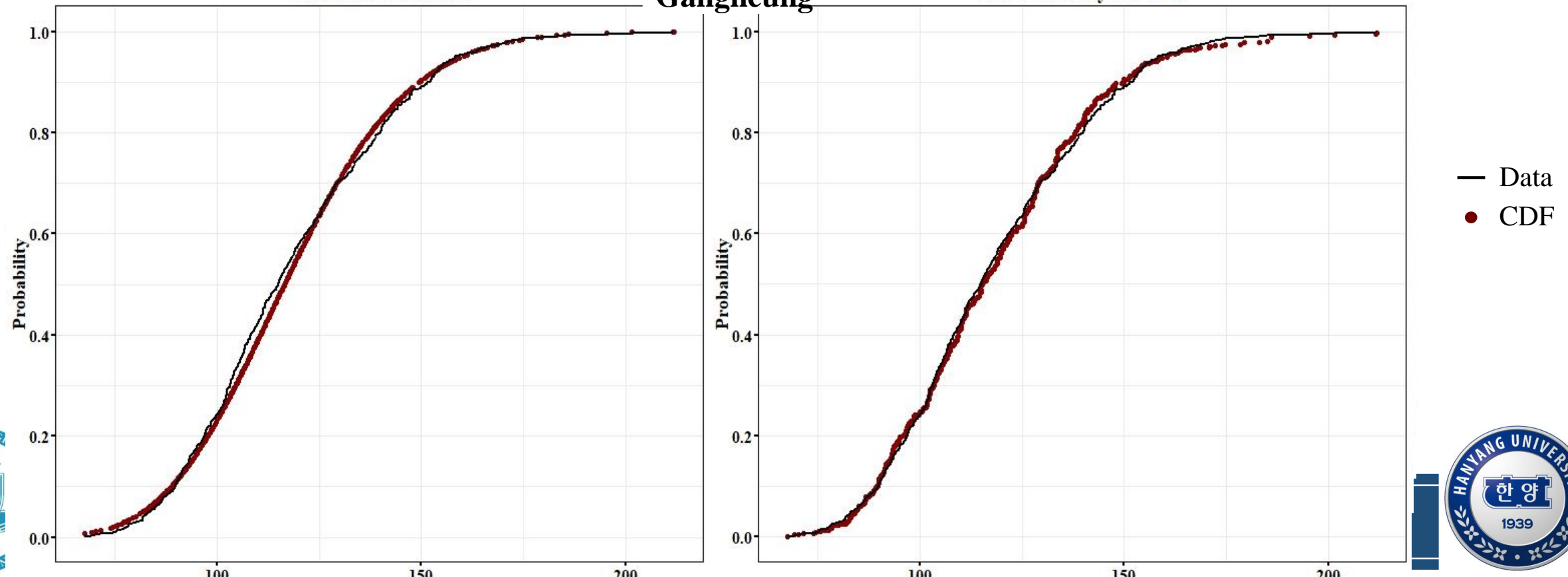
Non-Stationary SPI CDF

Conventional SPI CDF

Gangneung

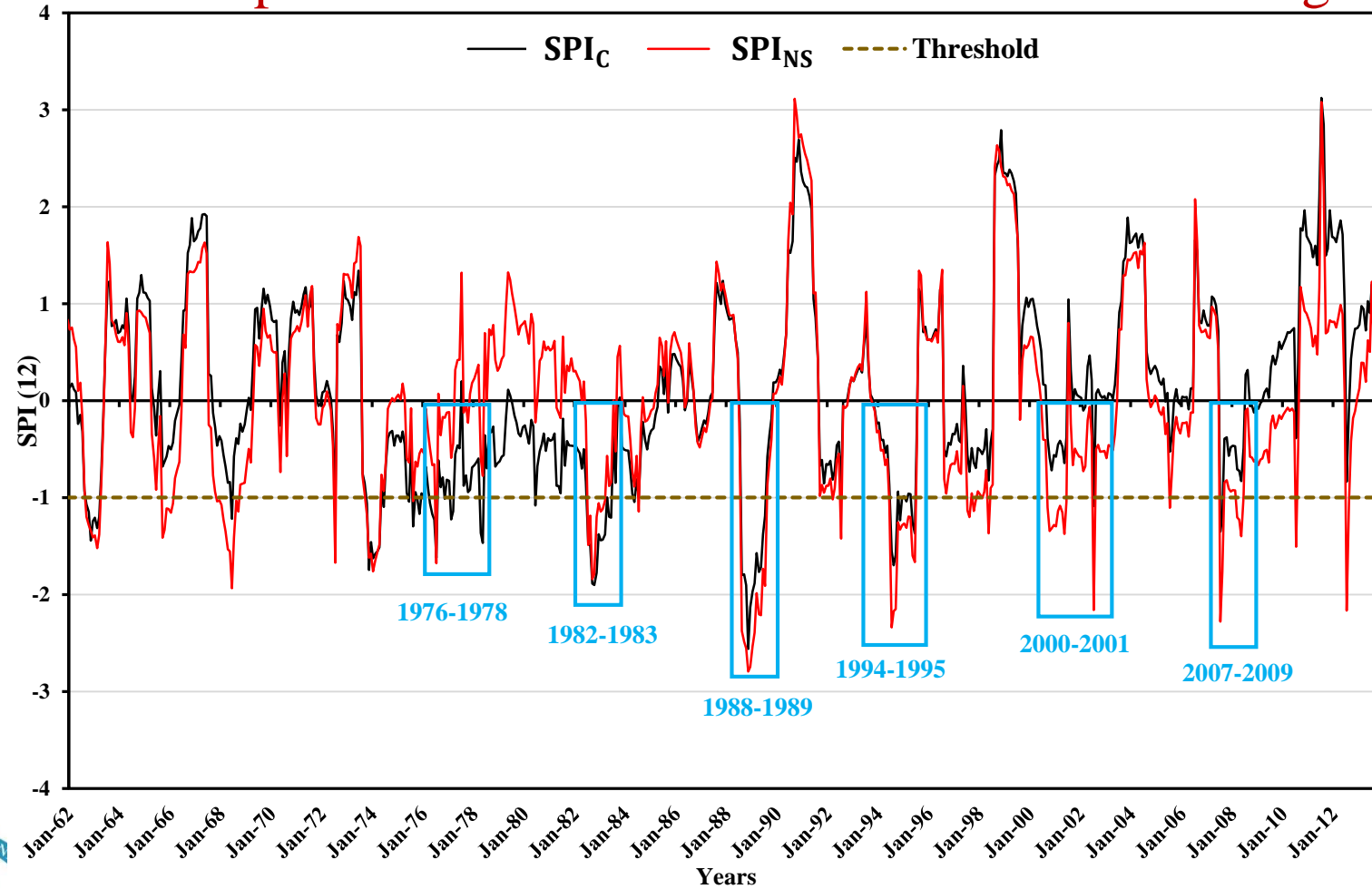
Non-Stationary SPI CDF

Conventional SPI CDF



# Results

## Comparison of $SPI_C$ & $SPI_{NS}$ with Historical Droughts at Seoul station



Seoul Station		
Drought Characteristics	$SPI_C$	$SPI_{NS}$
Severity	6.066	8.388
Duration	4.357	5.8125
Frequency	14	16
Inter-Arrival time	41.46	39.86

(Lee and Kim 2013)

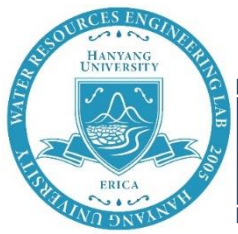


# Results

- Drought characteristics (Duration & Severity) extracted from SPI<sub>NS</sub> were fitted to several marginal distributions and best distribution was selected based on AIC criteria. Results of AIC criteria are shown below.

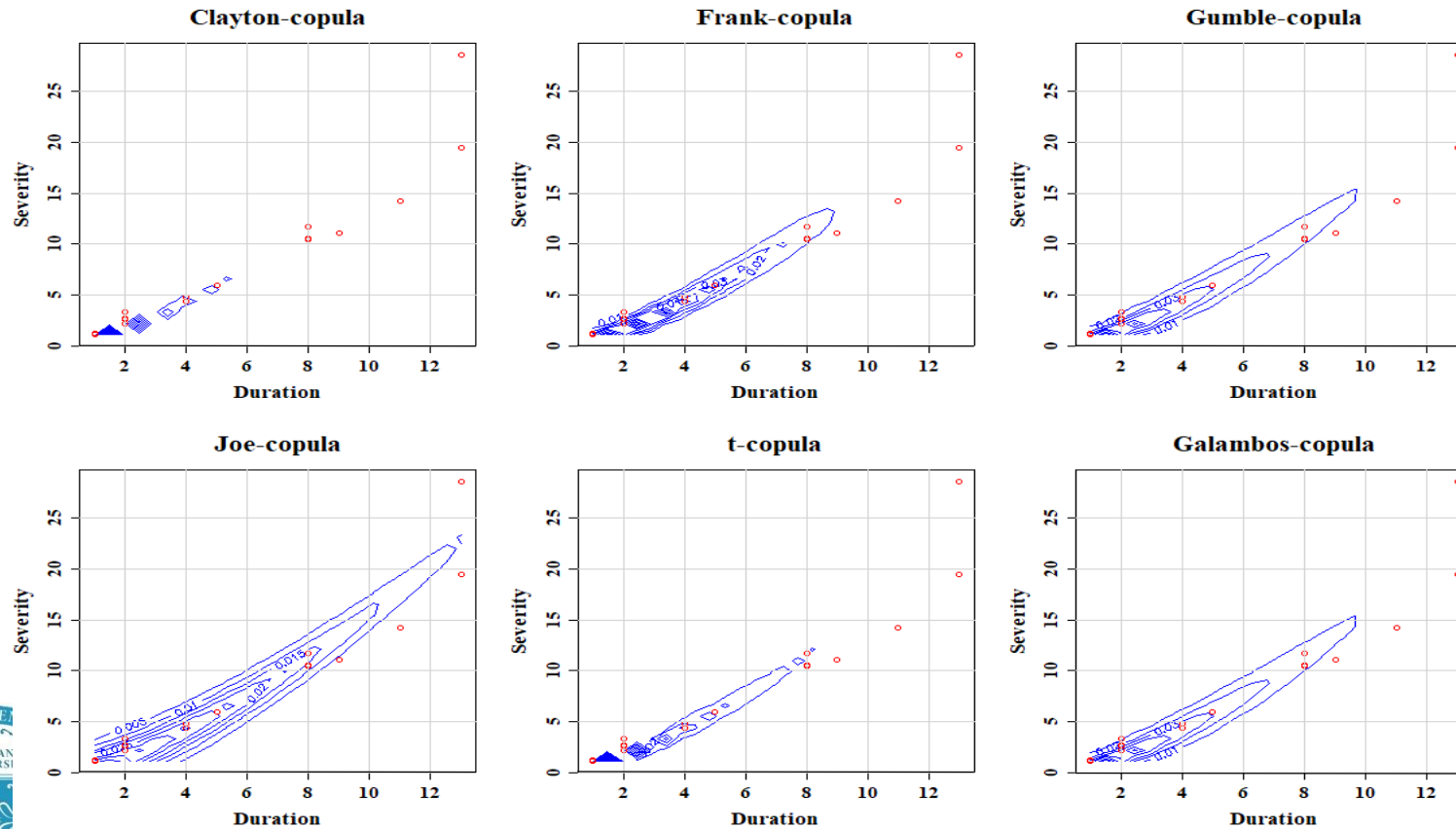
AIC Criteria for Duration					
Stations	Exponential	Gamma	Log-Normal	Log-Logistic	Weibull
Seoul	90.32	89.84	90.58	92.12	89.73
Chupungryung	93.58	93.8	95.96	97.58	93.11
Gangneung	88.65	87.46	88.32	89.59	87.44
Gwangju	85.07	82.69	83.69	84.93	82.4
Jeonju	78.59	76.64	78.91	79.44	75.5
Pohang	102.5	95.82	96.28	98.44	95.61
Ulsan	110.55	109.84	108.27	109.65	110.65
Yeosu	104.32	102.7	103.28	105.05	102.74

AIC Criteria for Severity					
Stations	Exponential	Gamma	Log-Normal	Log-Logistic	Weibull
Seoul	102.06	103.17	102.82	104.07	103.39
Chupungryung	107.44	108.95	111.17	112.75	108.62
Gangneung	102.63	103.41	104.1	105.3	103.47
Gwangju	97.69	98.52	98.31	99.57	98.72
Jeonju	89.61	90.2	91.62	92.36	90.002
Pohang	114.48	111.58	111.77	113.94	111.59
Ulsan	125.08	126.08	124.51	125.9	126.49
Yeosu	120.46	121.042	120.68	122.106	121.357



# Results

- Several copula functions (Clayton, Frank, Gumbel, Joe, t and Galambos) were tested to construct linkage between drought duration and drought severity.
- Best copula was selected based on p-value of Cramer-von Mises test. The results of all the copula functions in Seoul station are shown in Figure below.



Best Copula		
Stations	Name	Parameter
Seoul	Frank	32.949
Chupungryung	Clayton	17.641
Gangneung	Clayton	22.506
Gwangju	Frank	29.126
Jeonju	Frank	23.049
Pohang	Frank	17.725
Ulsan	Gumbel	7.7404
Yeosu	Frank	24.985

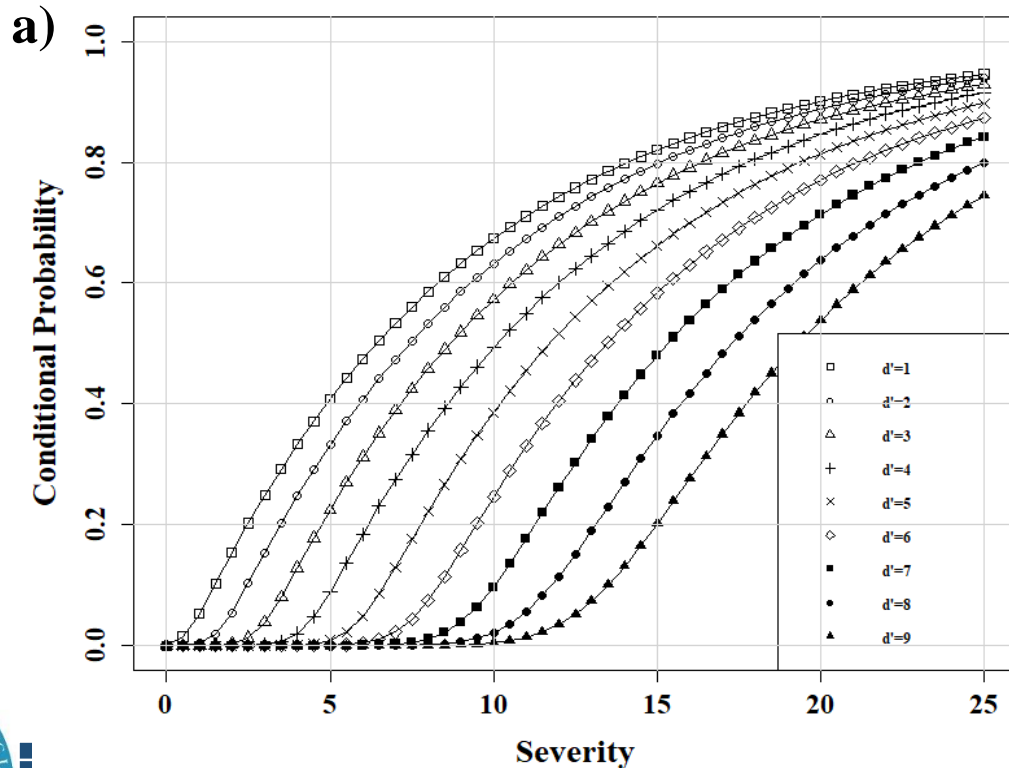


# Results

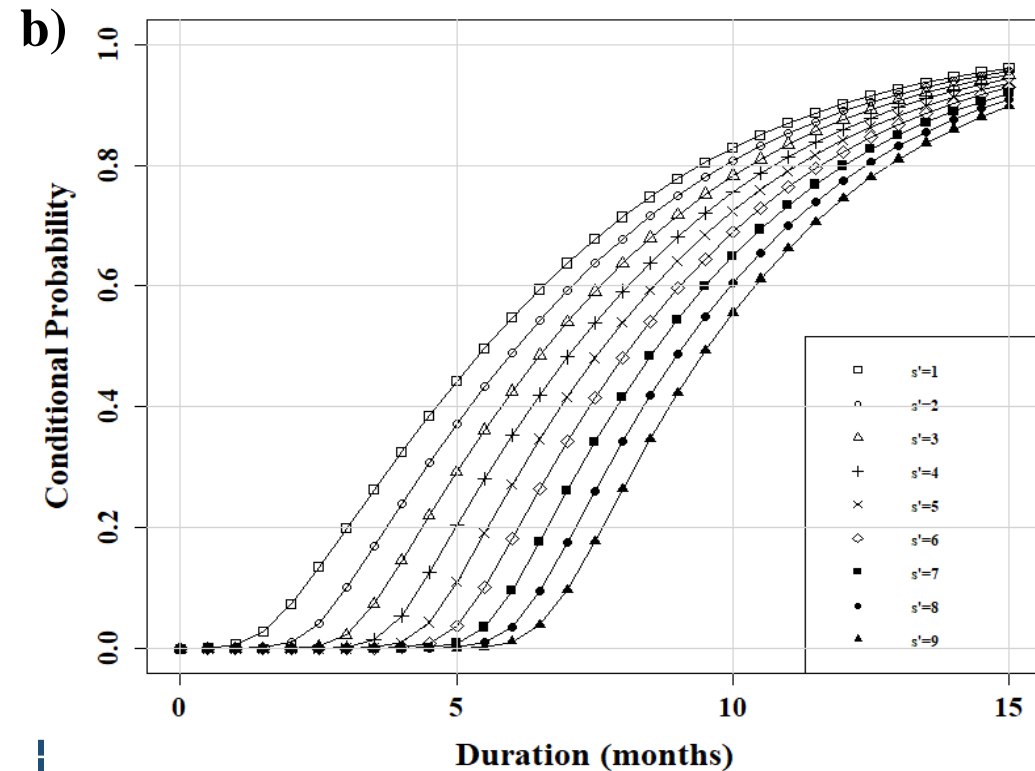
## Conditional Drought Distribution

- Results of conditional distribution of drought severity given drought duration exceeding a certain threshold  $d'$  is shown in Fig. a, whereas conditional distribution of drought duration given drought severity exceeding a certain threshold  $s'$  is shown in Fig. b.

Conditional Probability for  $P(S \leq s | D \geq d')$



Conditional Probability for  $P(D \leq d | S \geq s')$



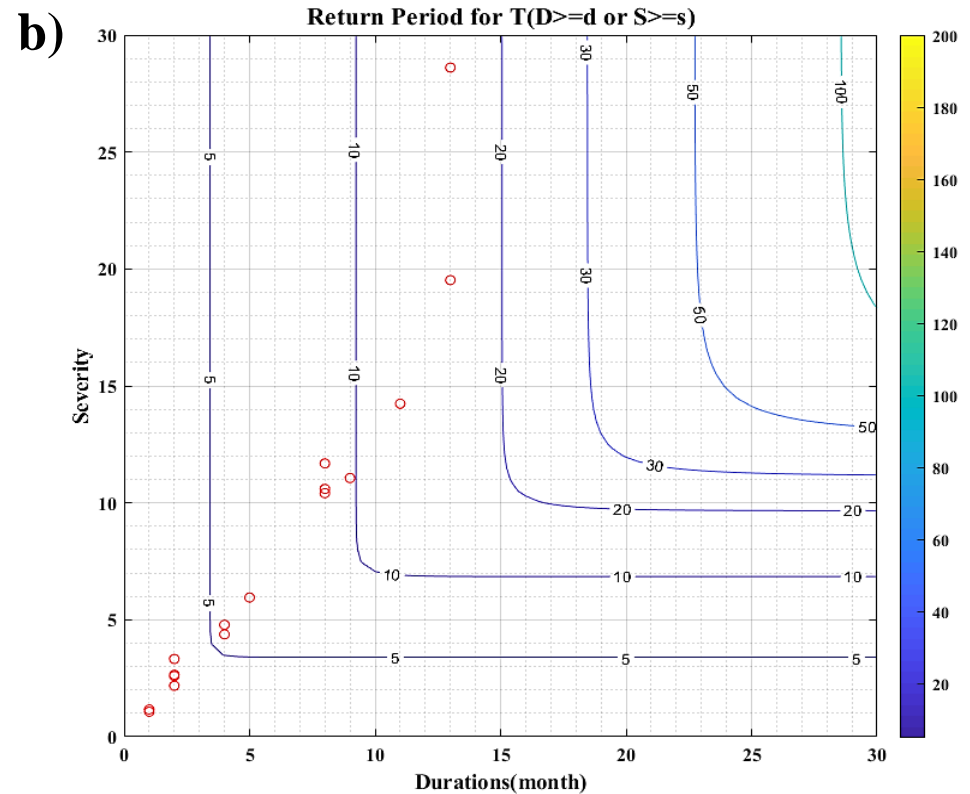
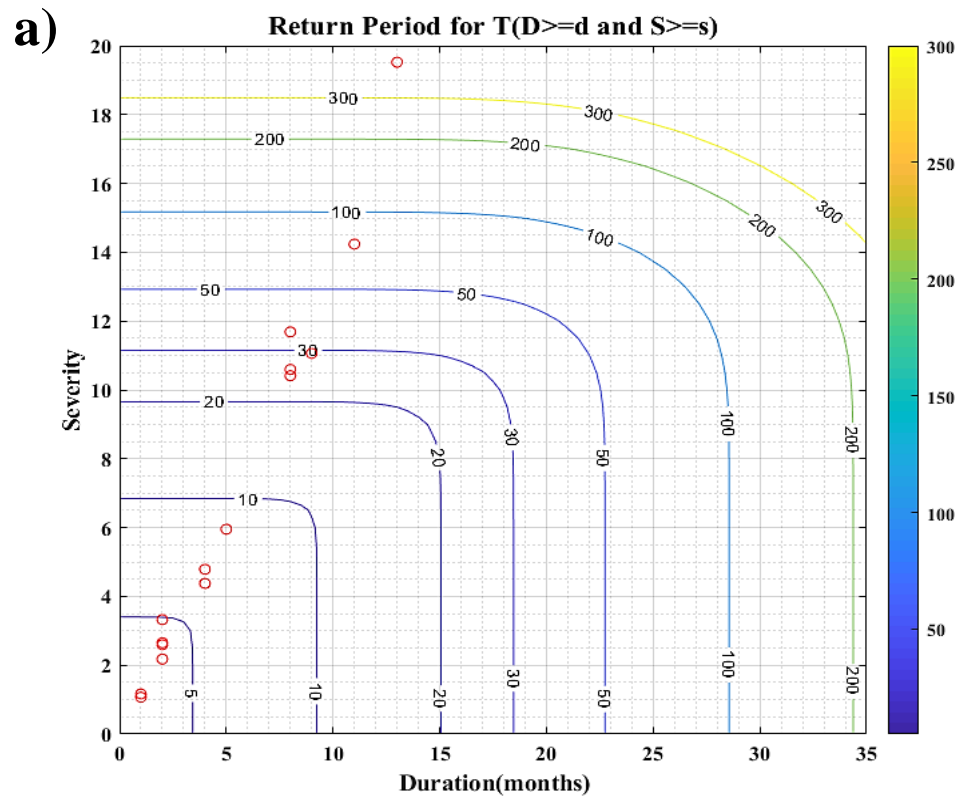
# Results

## Joint Return Period

- Results of joint return period between drought duration and severity is defined in two cases:

- i) return period of  $D \geq d$  and  $S \geq s$
- ii) return period of  $D \geq d$  or  $S \geq s$

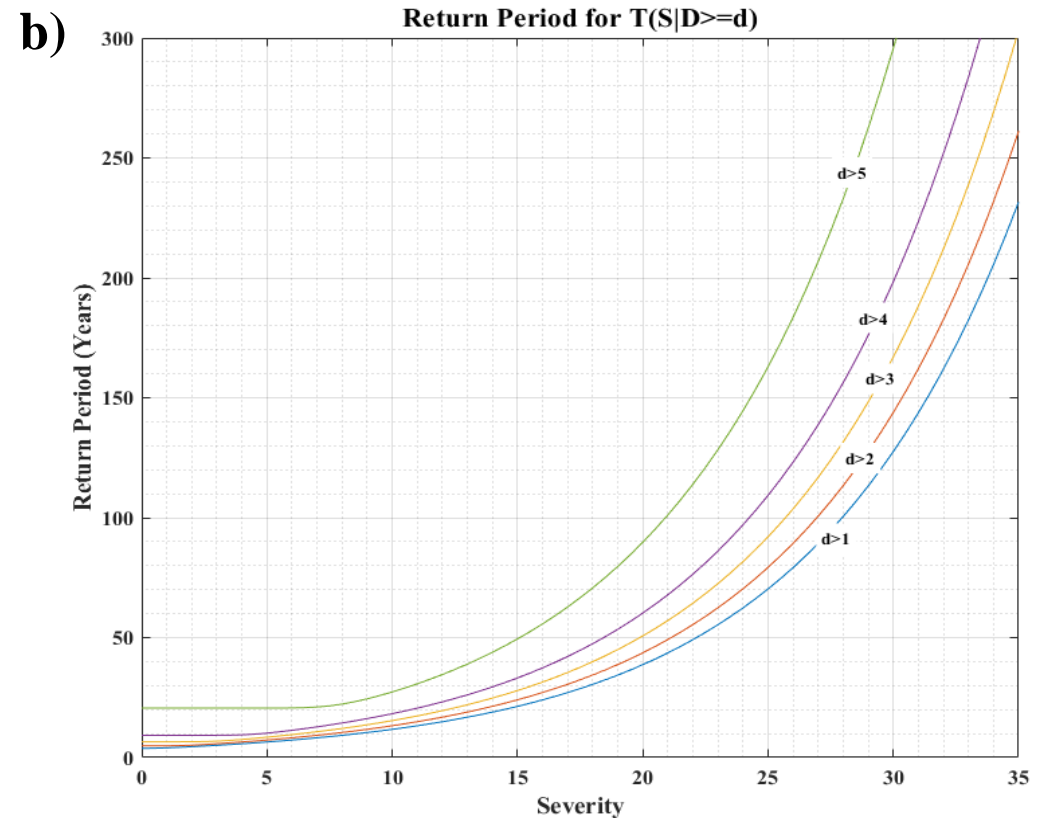
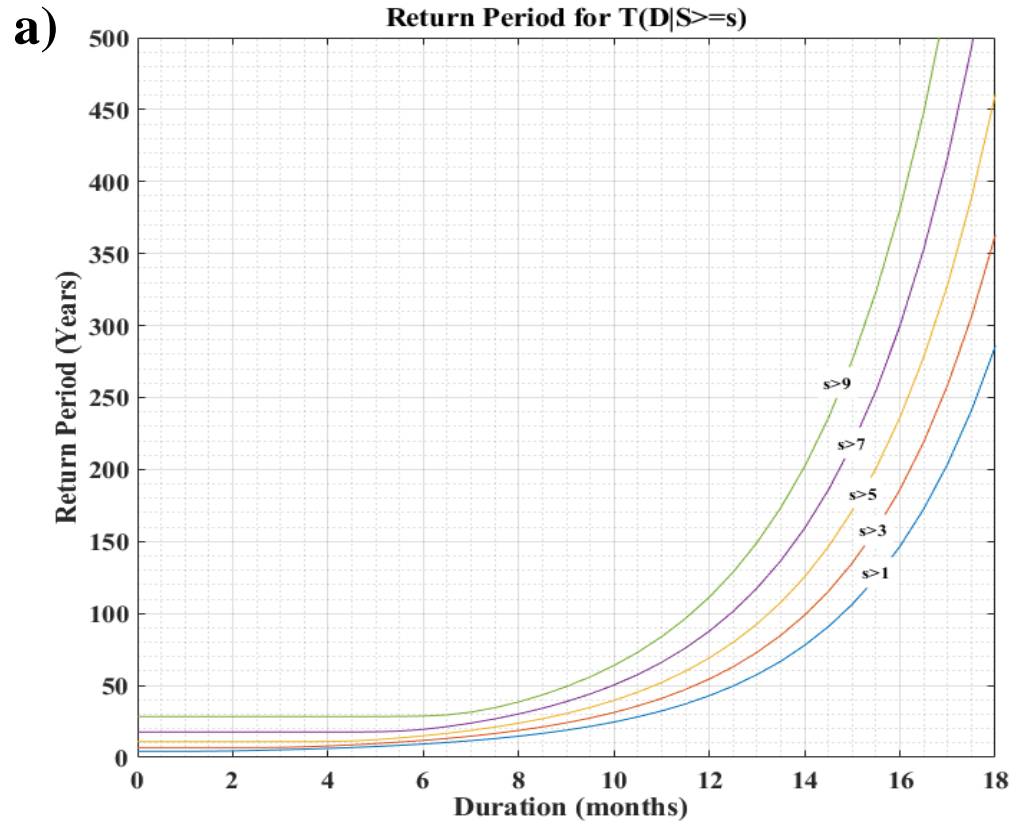
are shown in figures a and b. The return periods are demonstrated by contour lines.



# Results

## Conditional Return Period

- The results of conditional return period for drought duration given drought severity exceeding a certain threshold is shown in Fig. a., whereas the return period of drought severity given drought duration exceeding a certain threshold is shown in Fig. b.



# Results

## Univariate Return Period

SPI <sub>C</sub>		
Seoul Station	TD	Ts
Return Period	Drought duration	Drought Severity
(years)	(months)	
5	1.61	2.05
10	4.7	5.3
20	7.68	9.5
50	11.65	16.8
100	14.65	23.95

## Bivariate Return Period

Duration	Severity	TD and S	TD or S
(months)		(years)	(years)
2	2.5	6.15	4.93
5	6	13.86	9.36
8	10	35.23	15.78
11	14	91.89	24.39
13	19.5	335.8	31.93

## SPI<sub>NS</sub>

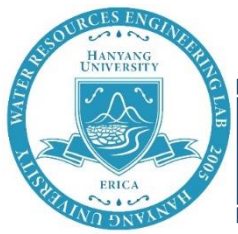
Seoul Station	TD	Ts
Return Period	Drought duration	Drought Severity
(years)	(months)	
5	3.4	3.5
10	6.86	9.3
20	9.7	15.2
50	12.92	22.8
100	15.2	28.55

Duration	Severity	TD and S	TD or S
(months)		(years)	(years)
2	2.5	4.4	4.15
5	6	8.28	6.03
8	10	21.84	8.62
11	14	69.13	12.33
13	19.5	427.49	15.65



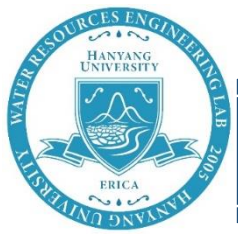
# Conclusions

- In this study, 53 years of precipitation records at 8 rain gauge stations were used to construct a non-stationary SPI<sub>NS</sub> at 12-month timescale.
- The performance of SPI<sub>NS</sub> were compared with conventional SPI<sub>C</sub>.
- The bivariate probabilistic properties of droughts such as joint probabilities, conditional probabilities, joint return periods and conditional return periods were considered for drought risk assessment among both drought indices.
- The findings suggested that SPI<sub>NS</sub> is capable for drought modeling and can be used for drought risk assessment.
- The drought characteristics (duration, severity & frequency) extracted from SPI<sub>NS</sub> were more severe than SPI<sub>C</sub> at Seoul station.
- The results of univariate return period shows that drought with a return period of 5 years will have high severity and longer duration in case of SPI<sub>NS</sub>.
- Similarly for bivariate analysis, a drought event with some duration and severity have shorter return period in case of SPI<sub>NS</sub>.
- These findings may be helpful for drought management and risk assessment under changing environment.



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*Thank You...*