IWRA's XVII

# WORLD WATER CONGRESS

## 제 17차 IWRA 세계물총회

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IWRA | DAEGU METROPOLITAN CITY | KOREA WATER RESOURCES ASSOCIATION | K water Korea Water Resources Corporation | Ministry of Environment

# Development of Two-Dimensional River Flow Analysis Model Using Godunovs Scheme and TVD Limiter

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# Plan

- **Part Ⅰ** : Provides an overview of the Shallow Water Equation(SWE) and the Riemann problem.
- **Part Ⅱ** : We check the application of the Riemann solver of the first-order accuracy method and the problem that occurs when the high-accuracy method is applied.
- **Part Ⅲ** : The model is verified through the application of the experimental channels example with actual experimental values.

# Shallow water wave equations

The shallow water wvae equations, given by

$$h_t + (uh)_x = 0$$

$$(uh)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = 0$$

Is an example of a system of equations written in conservative form.  More generally, we can write PDEs in conservative form as

$$q_t + (uh)_x = 0$$

These are typically derived form conservation laws for mass, momentum, energy, species, and so on.

Based on solving the conservative form of the shallow water wave equations using a finite volume method.

# Finite volume method

Assume a conservation law of the form

$$q_t + f(q)_x = 0$$

Define cell averages over the interval $C_i = [x_{i-1/2}, x_{i+1/2}]$

$$Q_i^n = \frac{1}{\Delta x} \int_{C_i} q(x, t_n)\, dx$$

How does the average evelve?

$$\frac{1}{dt} \int_{C_i} q(x,t)\, dx = -\int_{C_i} \frac{d}{dx} f\big(q(x,t)\big)$$
$$= f(q(x_{i-1/2}), t)) - f(q(x_{i+1/2}, t))$$

# Finite volume method

Evolution of the cell average value:

$$\frac{d}{dt}\int_{C_i} q(x,t)\,dx = f\left(q\left(x_{i-1/2},t\right)\right) - f\left(q\left(x_{i+1/2},t\right)\right)$$

**Integrate in time**

$$\int_{C_i} q(x,t_{n+1})\,dx = \int_{C_i} q(x,t_n)\,dx + \int_{t_n}^{t_{n+1}} \left[f\left(q\left(x_{i-1/2},t\right)\right) - f\left(q\left(x_{i+1/2},t\right)\right)\right] dt$$

# Finite volume method

Using numerical fluxes, we use the update formula:

$$Q_i^{n+1} = Q_i^n - \frac{\Delta t}{\Delta x}\left[F_{1+1/2}^n - F_{1-1/2}^n\right]$$

Written as

$$\frac{Q_i^{n+1} - Q_i^n}{\Delta t} - \frac{F_{1+1/2}^n - F_{1-1/2}^n}{\Delta t} = 0$$

this form resemble the conservation law:

$$q_t + f(q)_x = 0$$

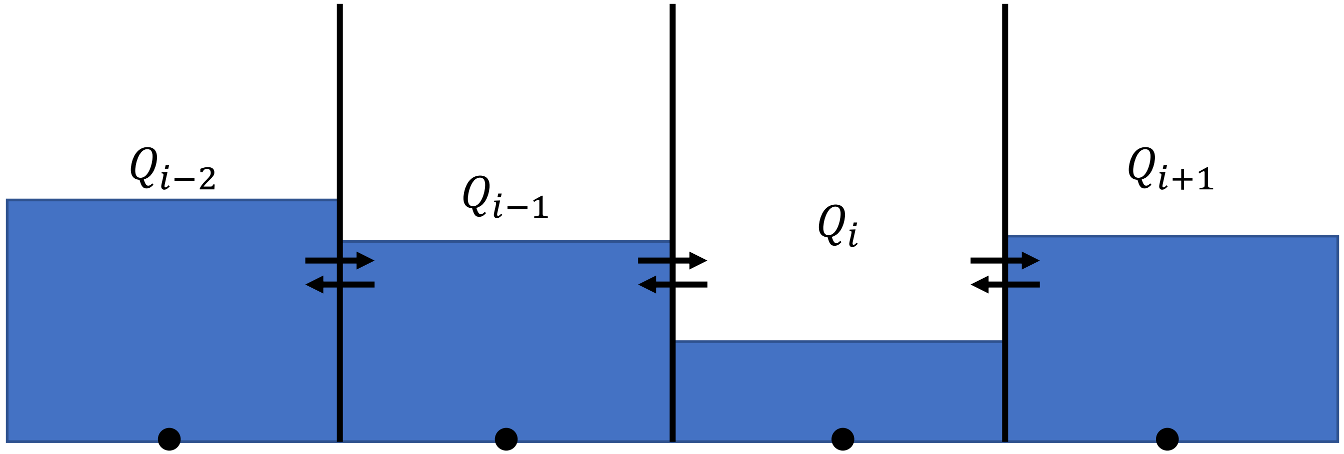
# Numerical fluxes

We want to approximate the numerical flux.

$$F_{i-1/2}^{n} \approx \frac{1}{\Delta t}\int_{t_n}^{t_{n+1}} f\left(q\left(x_{i-1/2}, t\right)\right) dt$$

For an explicit time stepping scheme, we try to find formulas for the flux of the form

$$F_{i-1/2}^{n} = \mathcal{F}(Q_i^n, Q_{i-1}^n)$$



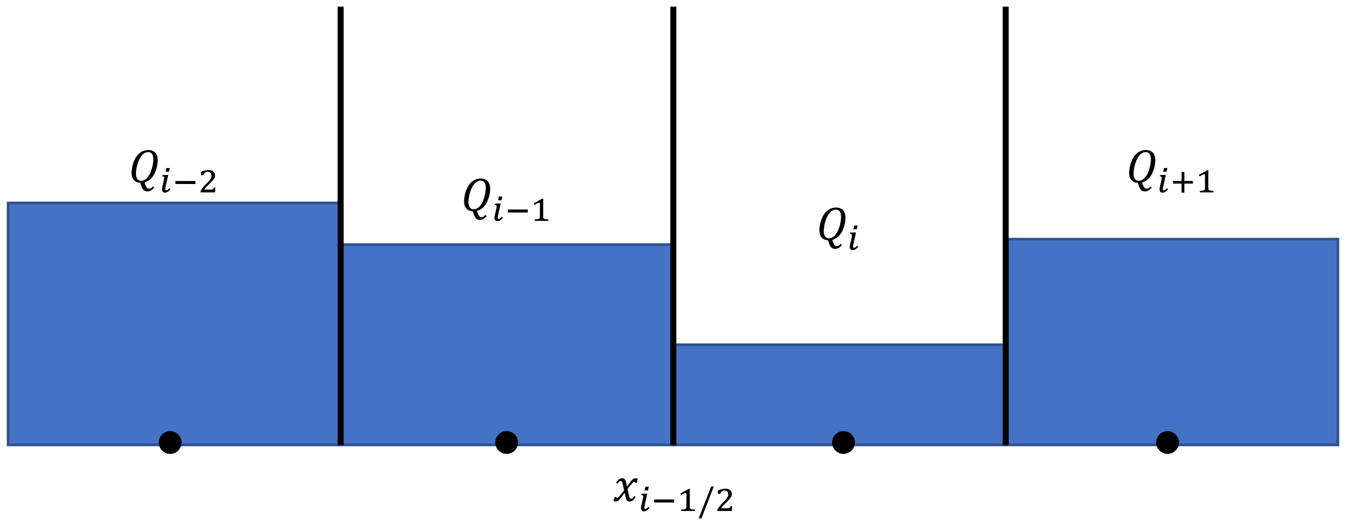
# Riemann problem

At each cell interface, solve the hyperbolic problem with special initial data, i.e.

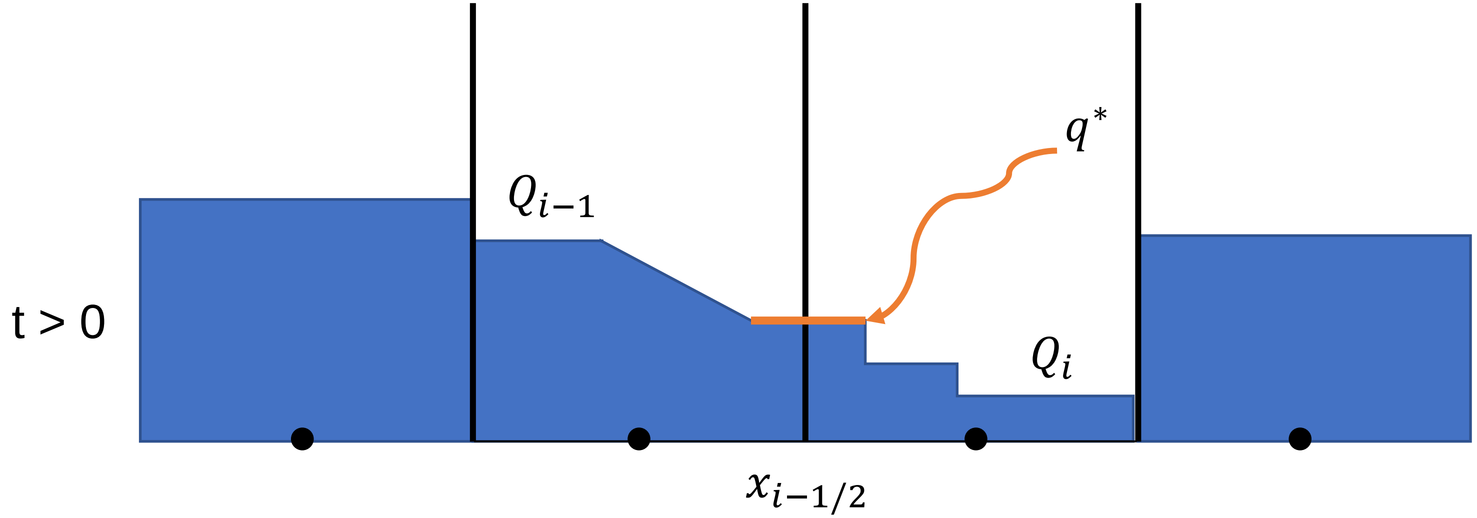
$$q_t + f(q)_x = 0$$

subject to

$$q(x,0) = \begin{cases} Q_{i-1} & x > x_{i-1/2} \\ Q_{i+1} & x < x_{i-1/2} \end{cases}$$



# Riemann problem




Numerical flux at cell interface is then approximated by

$$F_{i-1/2} = f(q^*)$$

This is the classical Godunov approach for solving hyperbolic conservation laws.

- Resolves shocks and rarefactions

# Using Riemann solvers

- **Lax–Friedrichs (first-order scheme)**

The Lax–Friedrichs method, named after Peter Lax and Kurt O. Friedrichs, is a numerical method for the solution of hyperbolic partial differential equations based on finite differences. One can view the Lax–Friedrichs method as an **alternative to Godunov's scheme**, where one avoids solving a Riemann problem at each cell interface, at the expense of adding **artificial viscosity**.

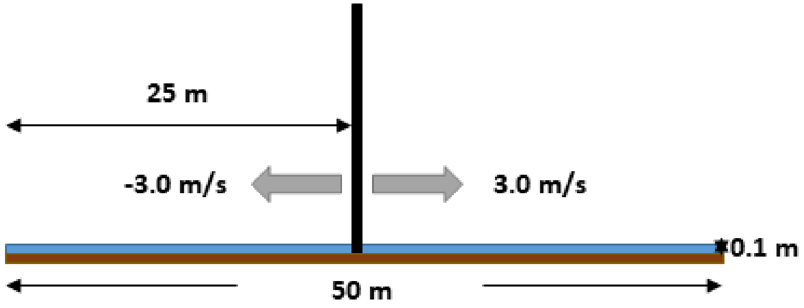
- **HLLC (first-order scheme**)

The HLLC (Harten-Lax-van Leer-Contact) solver was introduced by Toro. It restores the missing Rarefaction wave by some estimates, like linearisations, these can be simple but also more advanced exists like using the Roe average velocity for the middle wave speed. They are quite **robust** and efficient but somewhat **more diffusive.**

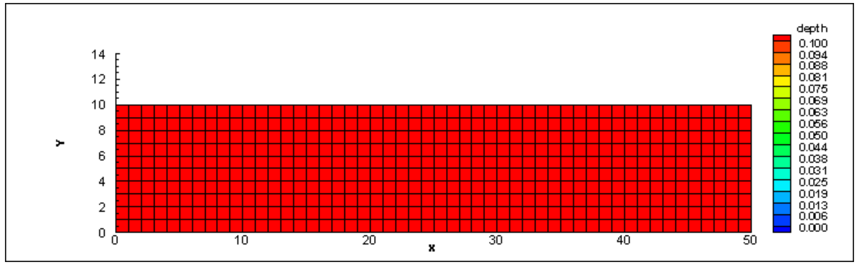
- **MUSCL Hancock TVD** (**Higher-order scheme**)

# Using Riemann solvers

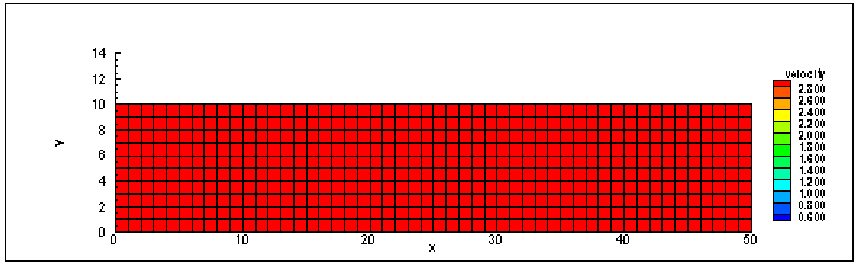
- **Generation of a dry bed**



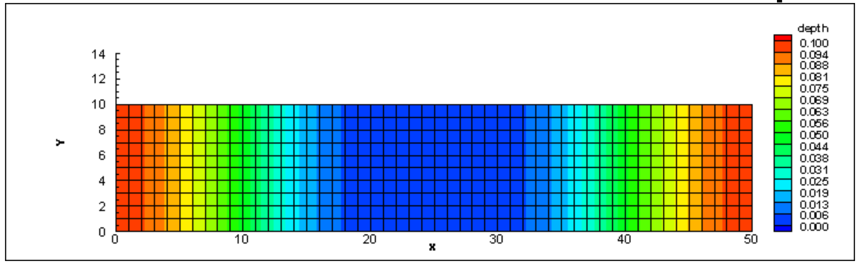

**t=0 sec** **depth**



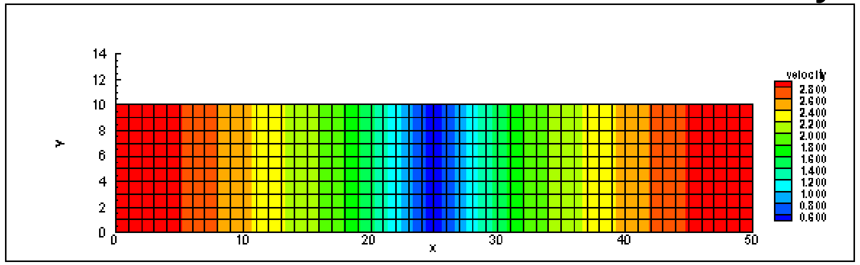

**velocity**



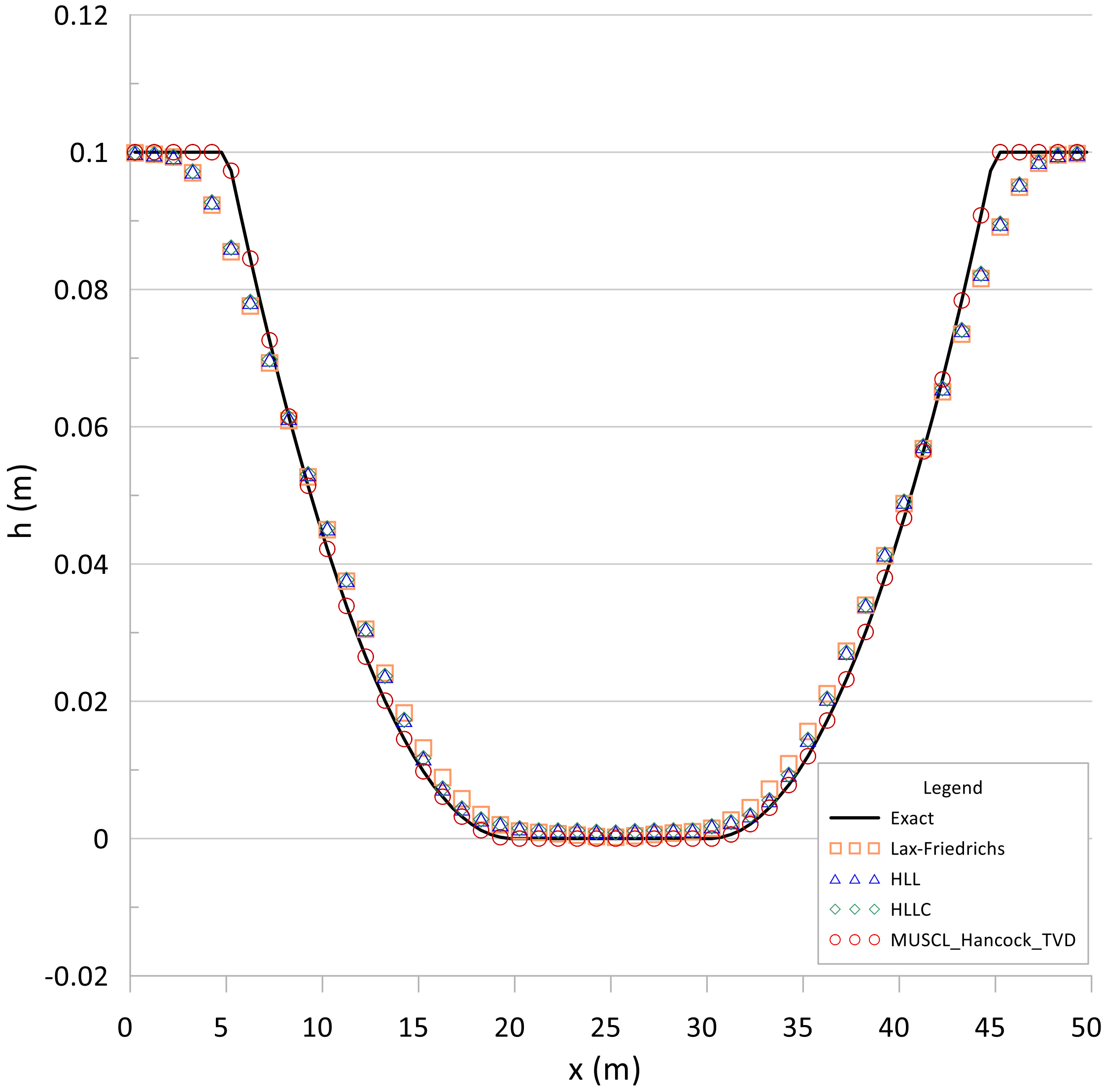

**t=5.0 sec** **depth**

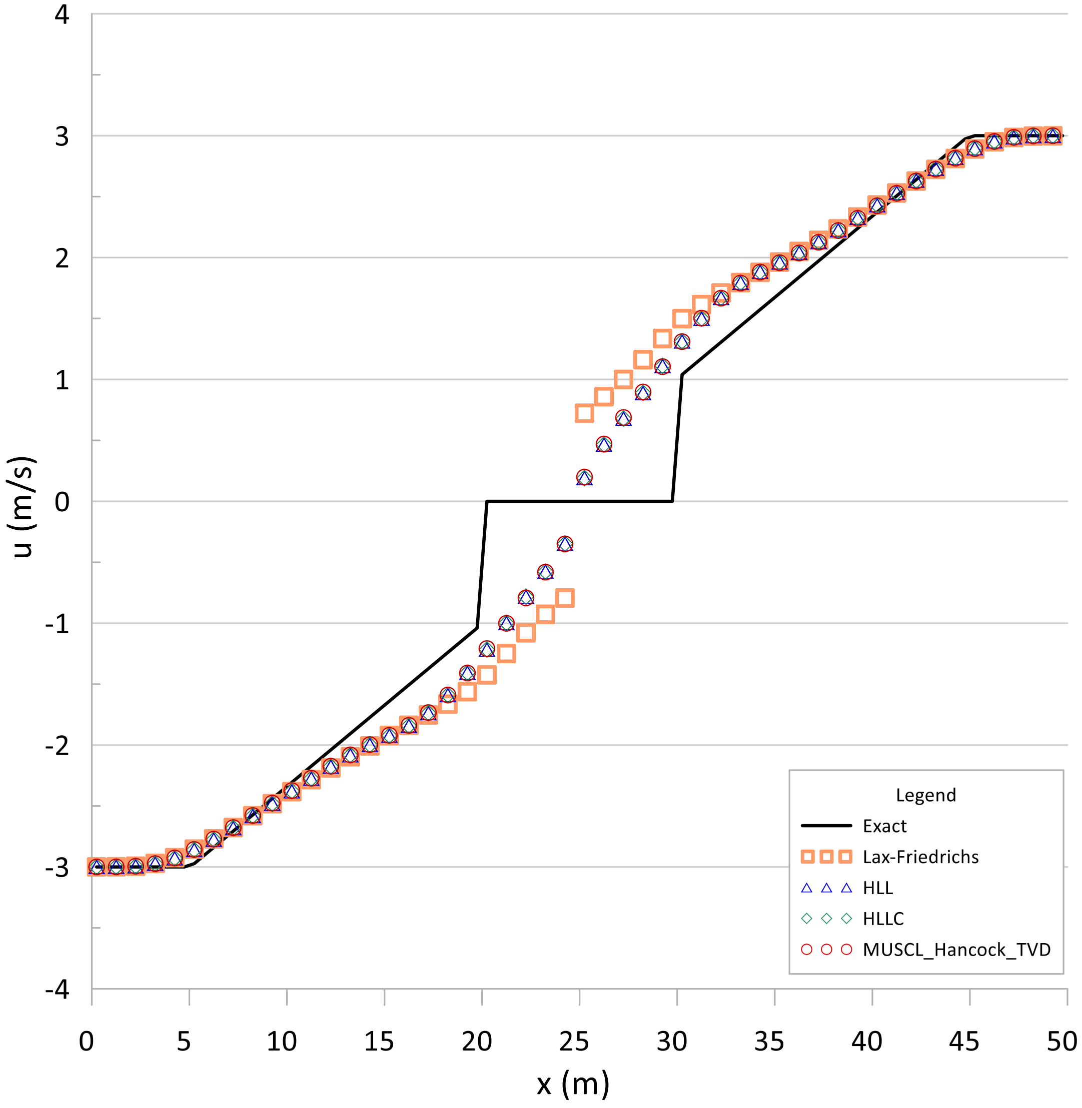



**velocity**



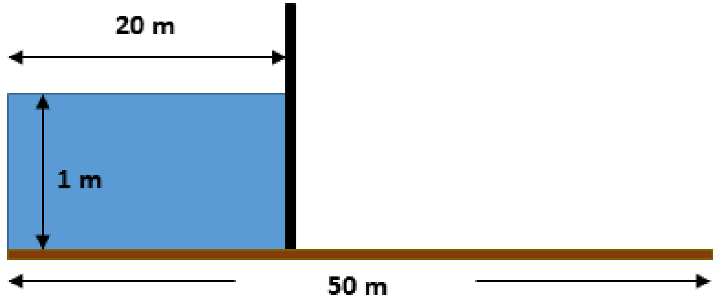
# Using Riemann solvers



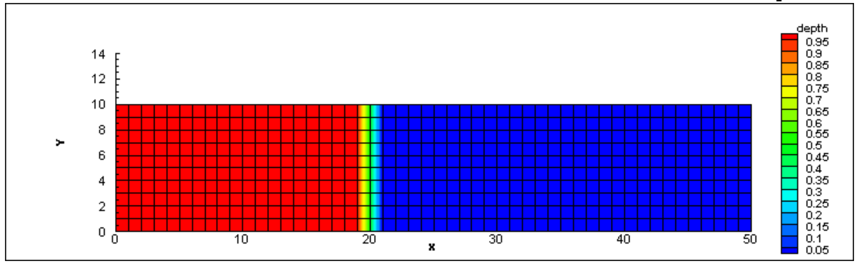
# Using Riemann solvers

- **Dambreak on dry bed**

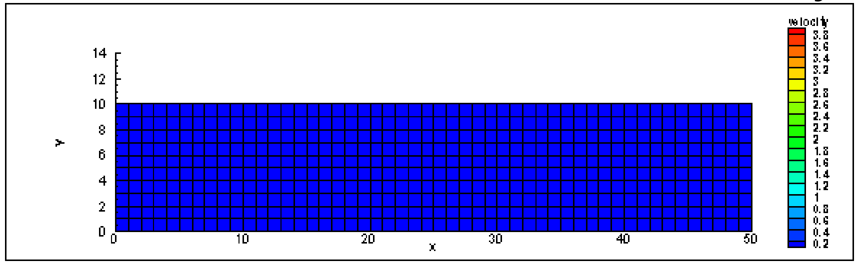



t=0 sec

depth

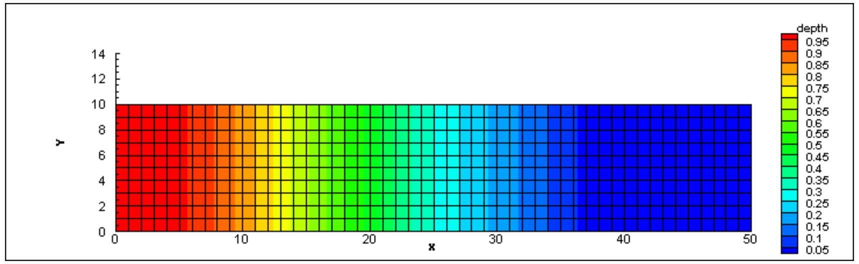


velocity

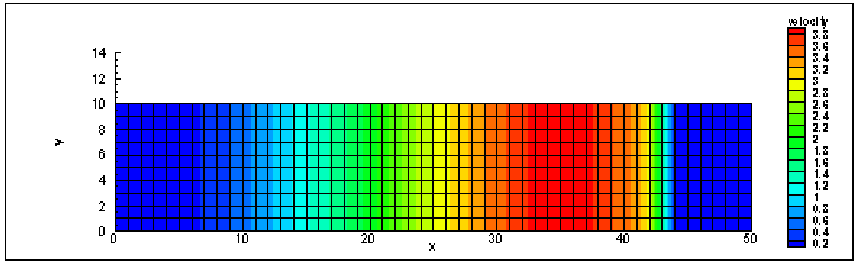


t=4.0 sec

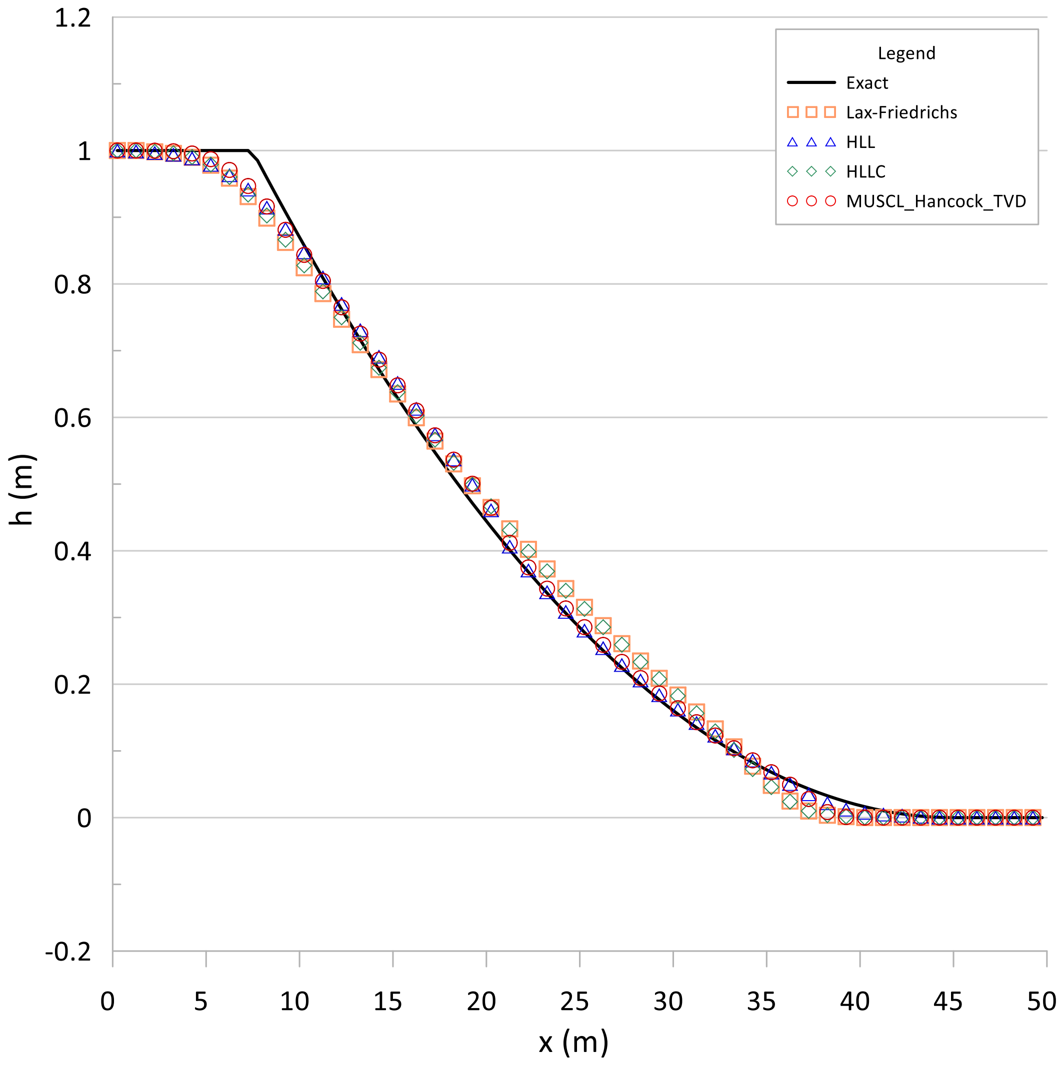
depth

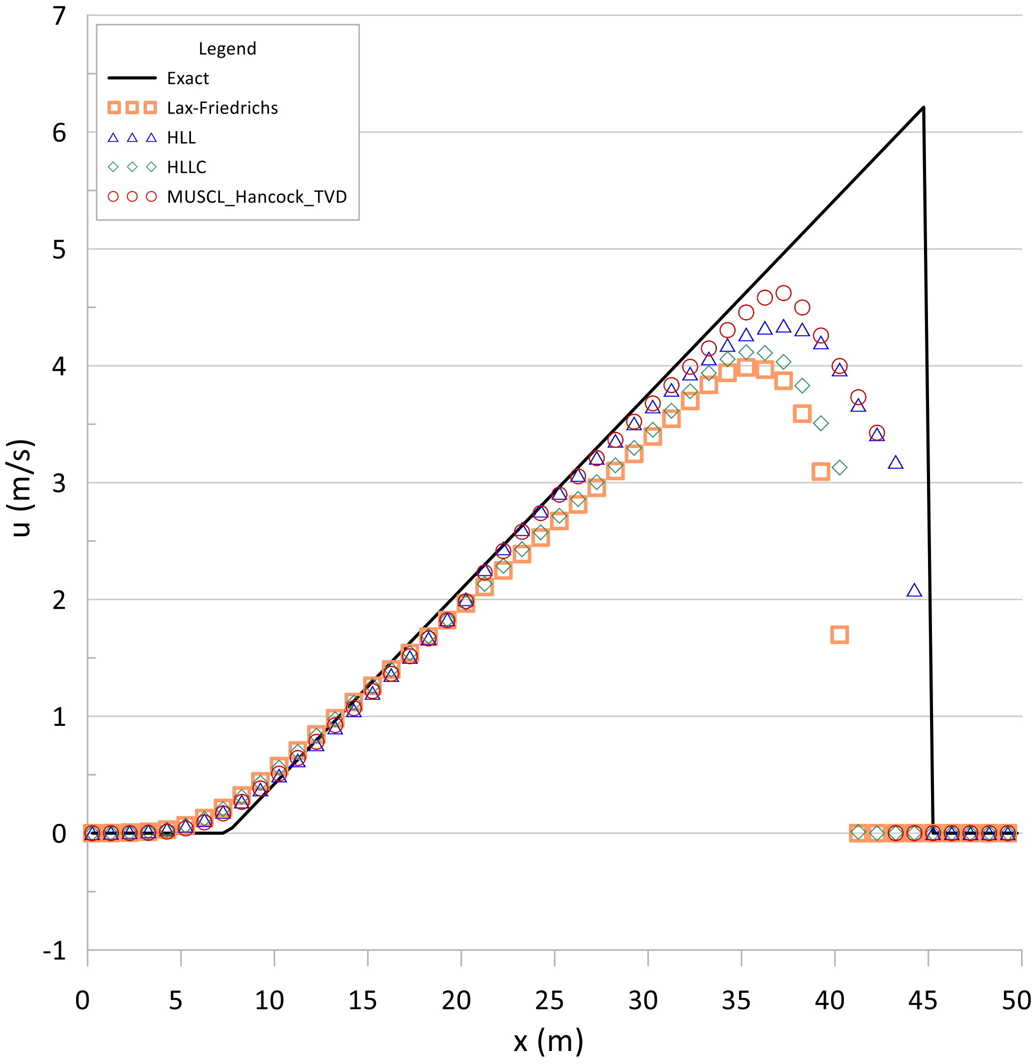


velocity



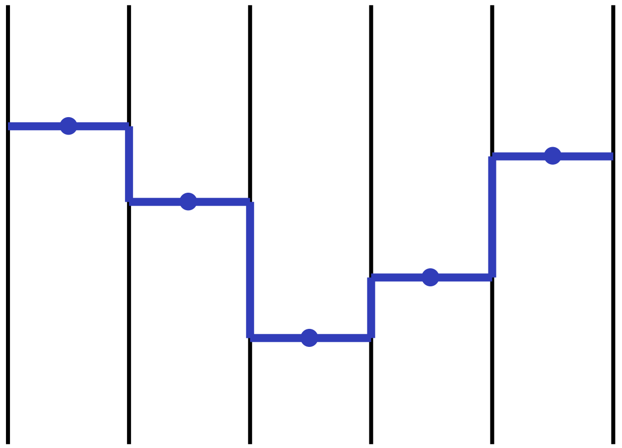
# Using Riemann solvers



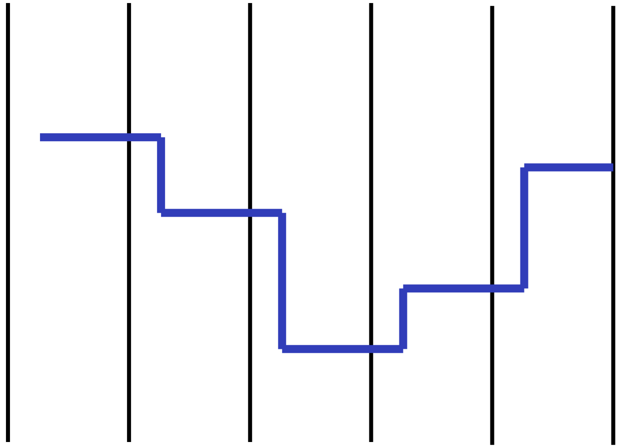



# Using Riemann solvers

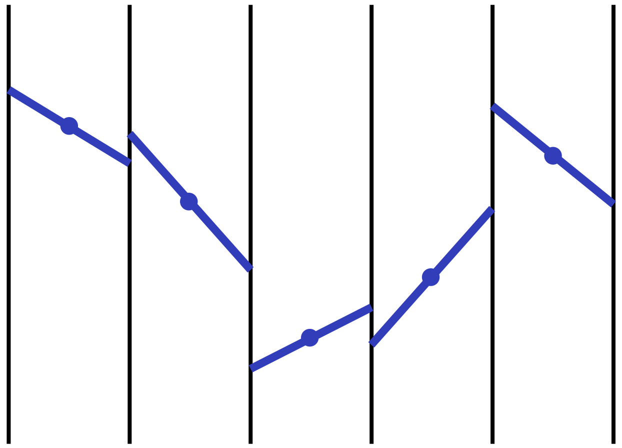
Cell averages and piecewise constant reconstruction:



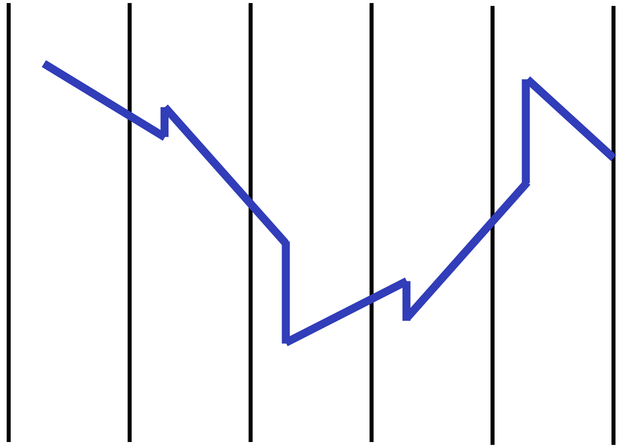
After evelution:



Cell averages and piecewise liner reconstruction:

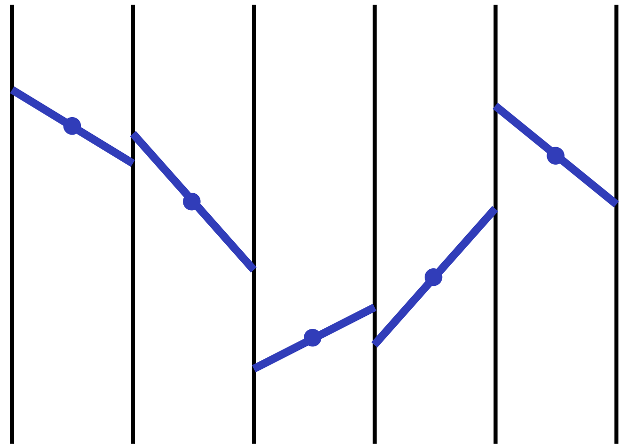


After evelution:



# Using Riemann solvers

Cell averages and piecewise liner reconstruction:



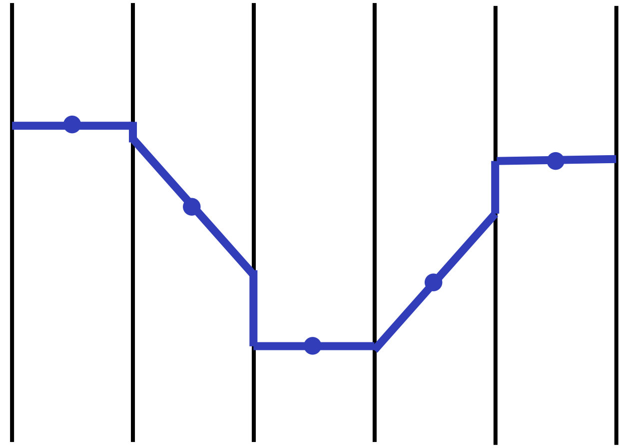
Want to use slope where solution is smooth for “second-order” accuracy

Where solution is not smooth, adding slope corrections gives oscillations.

Limit the slope based on the behavior of the solution.

$$\sigma_i^n = \left(\frac{Q_{i+1}^n - Q_i^n}{\Delta x}\right)\Phi_i^n$$

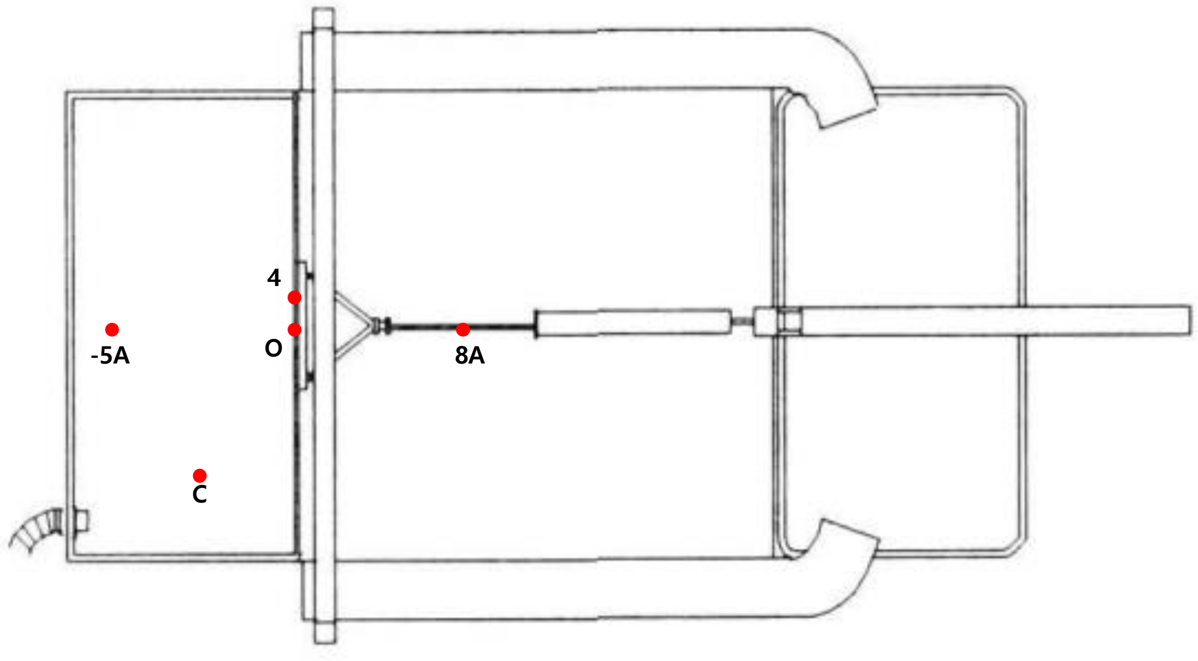
After evelution:



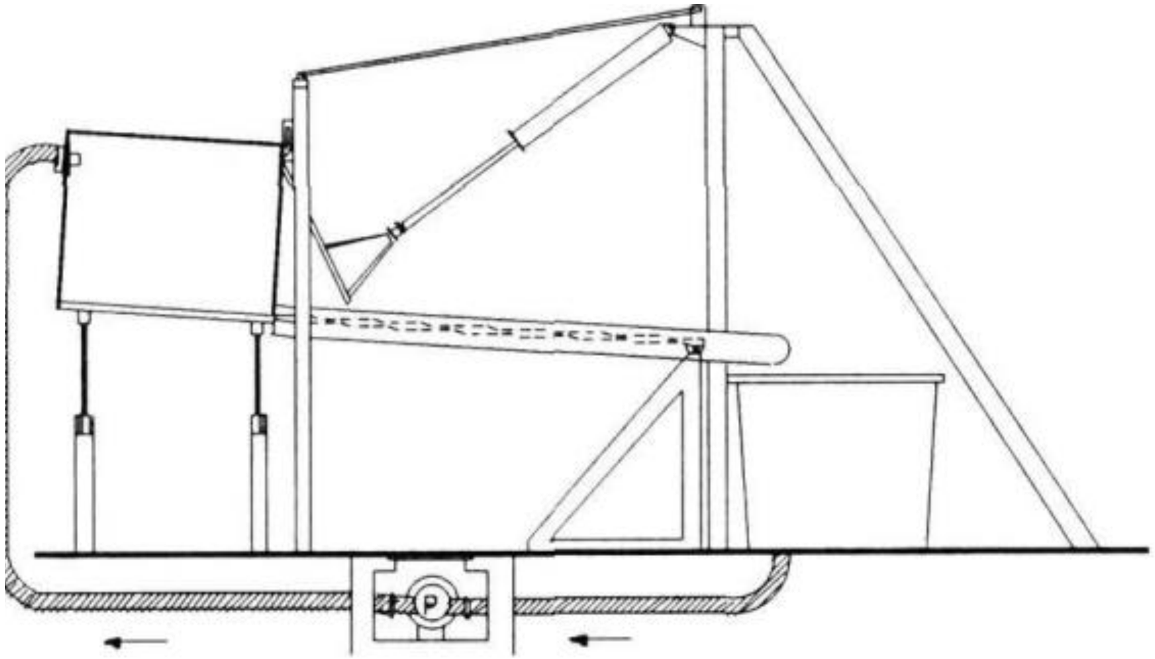
# Application to experimental channel

As a dam collapse experiment performed by Fraccarollo and Toro (1995), it was evaluated as an example to evaluate the shock wave generated during dam collapse and the numerical instability generated in a dry channel, and is used as a verification example in many studies.

**Top view**




**Side view**



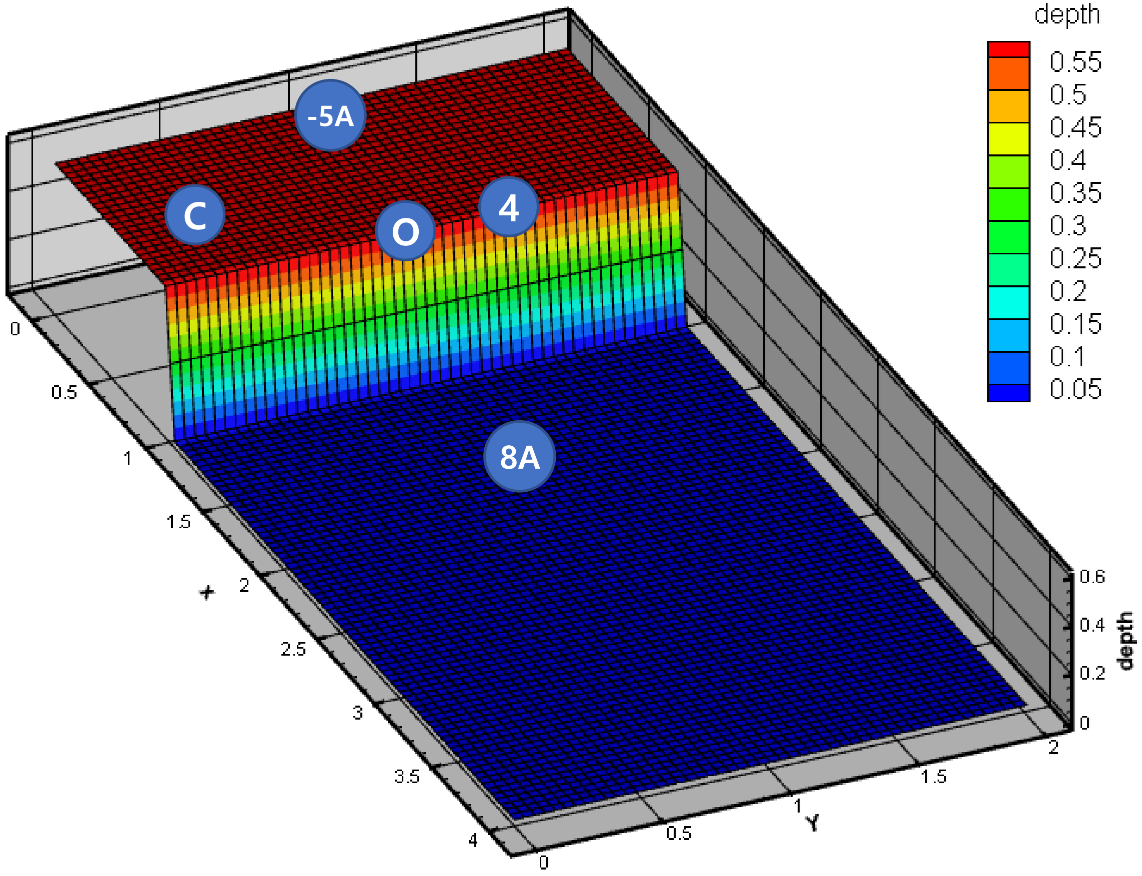
# Application to experimental channel

< Modeling condition>

- Size : 4m x 2m
- Upstream depth : 0.6m
- Downstream depth : dry state
- Upstream boundary
  : closed boundary
- Downstream boundary
  : free boundary

< Observation point >

-5A : x = 0.18m, y =1.00m
C : x = 0.48m, y = 0.40m
4 : x = 1.00m, y = 1.16m
O : x = 1.00m, y = 1.00m
8A : x = 1.722m, y = 1.00m



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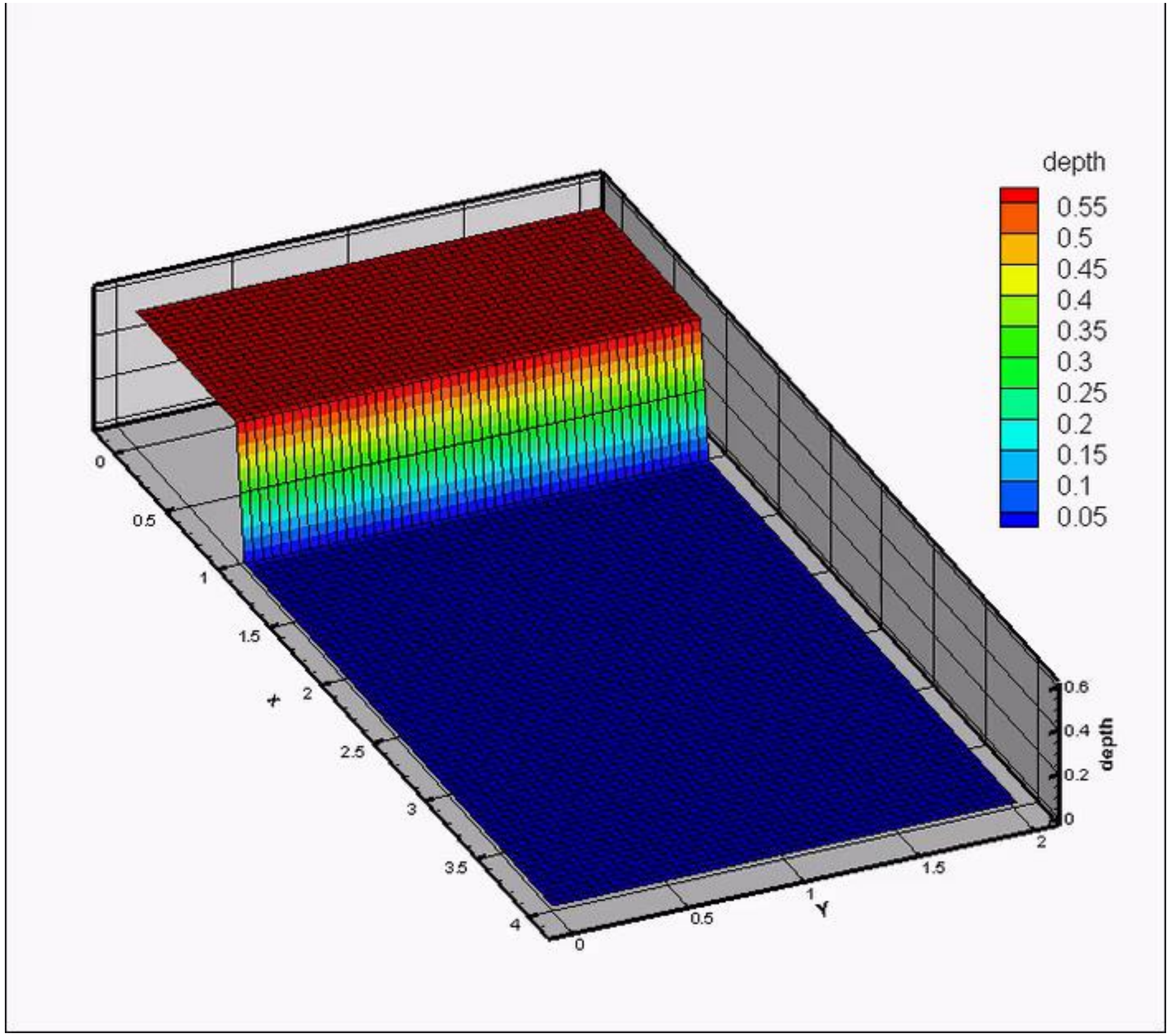


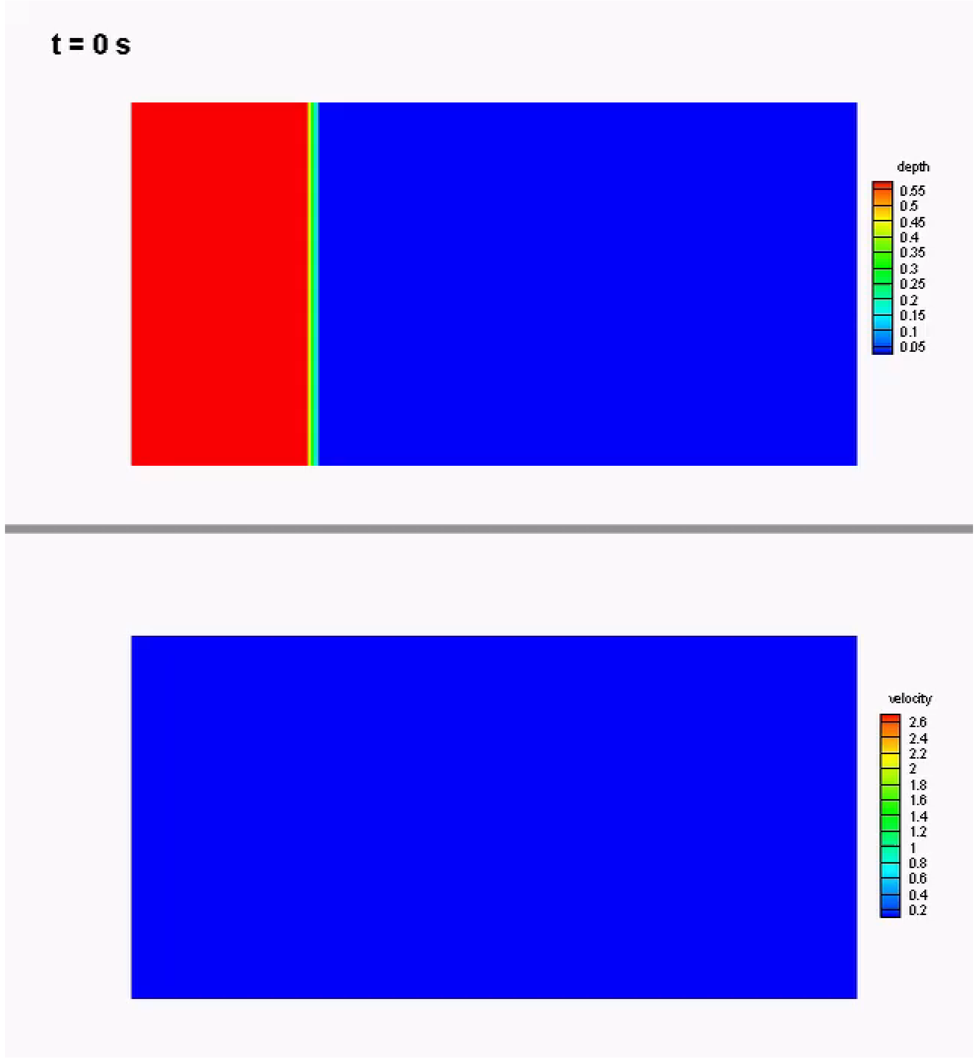

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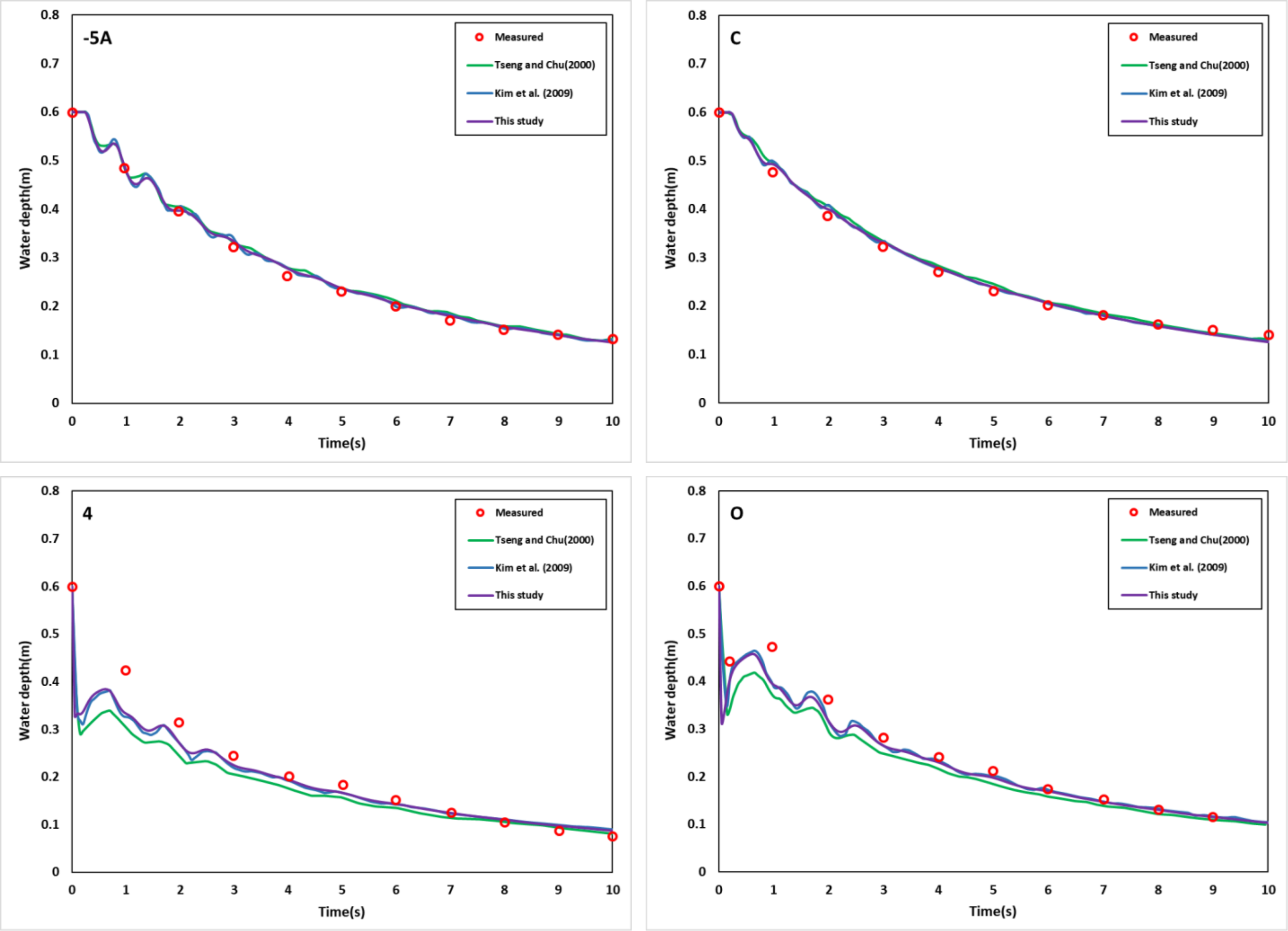
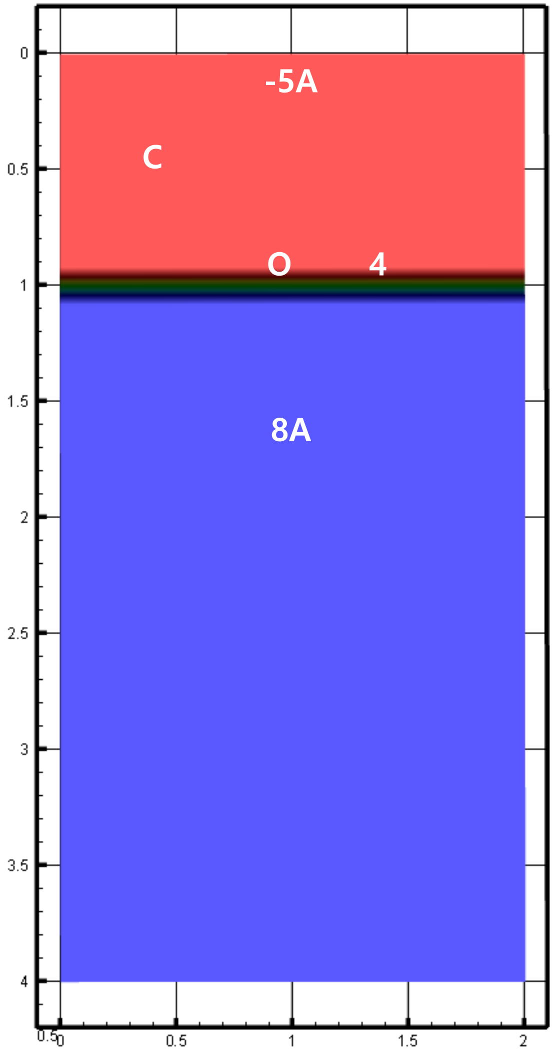
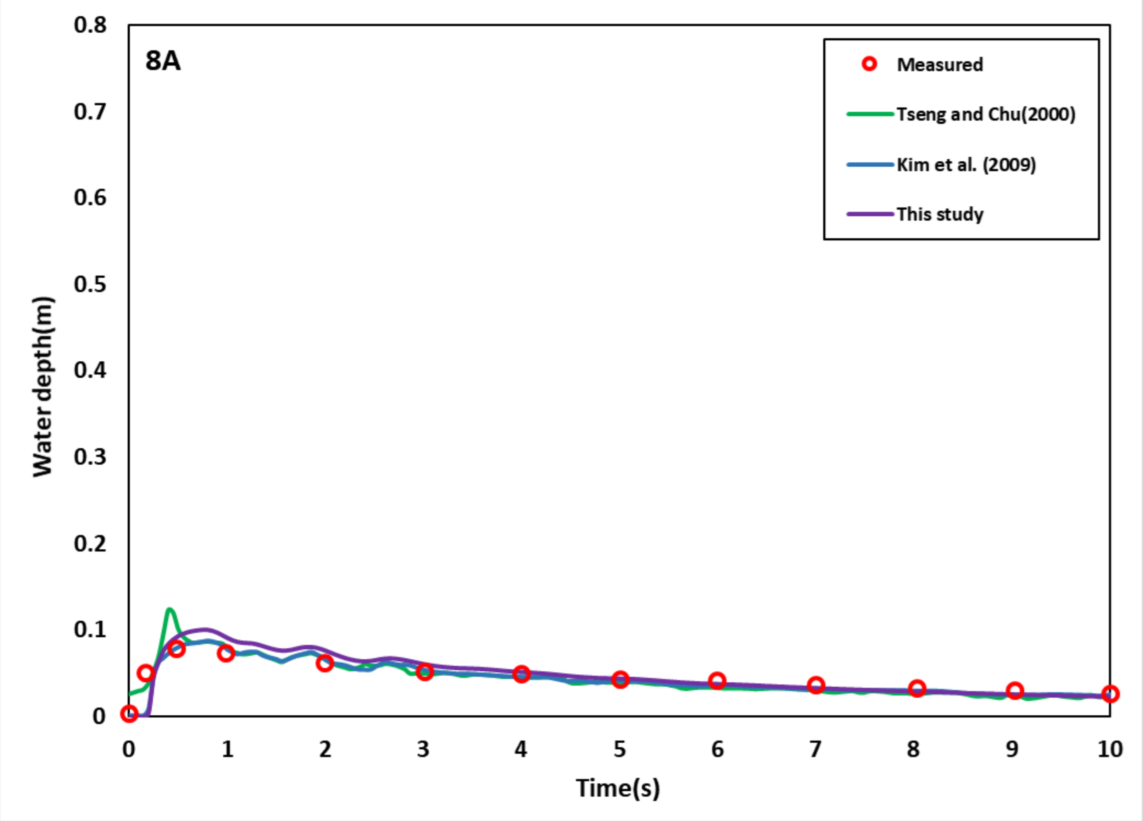


# Application to experimental channel




| RMSE(Root Mean Square Error) | | | |
|---|---|---|---|
| POINT | Tseng and Chu(2000) | Kim et al. (2009) | Simulated |
| -5A | 0.0088 | 0.0084 | 0.0073 |
| C | 0.0110 | 0.0111 | 0.0089 |
| 4 | 0.0465 | 0.0352 | 0.0331 |
| O | 0.0773 | 0.0645 | 0.0648 |
| 8A | 0.0176 | 0.0144 | 0.0155 |

- The flow prediction results in the upstream reservoir were high, and it was confirmed that there was no significant difference in accuracy for other sections as compared with the results of other simulations.

# THANK YOU