

# Non-stationary GEV Modeling of Precipitation Extremes

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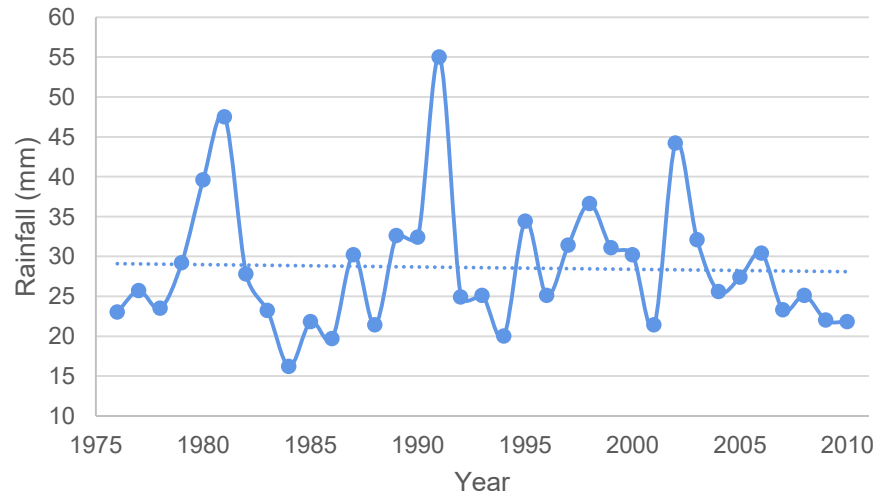
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# Stationarity Concept

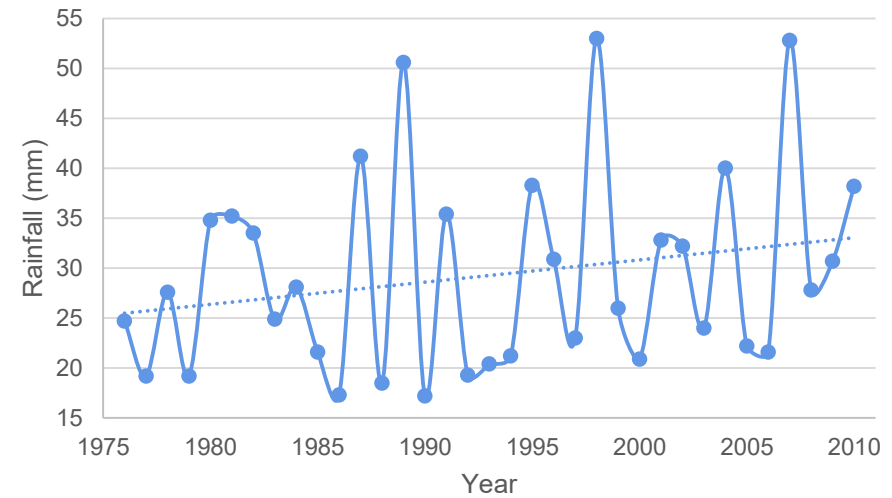
## Stationary

- Weak stationary
- Strict stationary



## Non-stationary

Changing statistics



# Study Area

Study area consists of 53 meteorological stations (MS) from generally central and southern Turkey. Time range for the study is determined as 1976-2005.

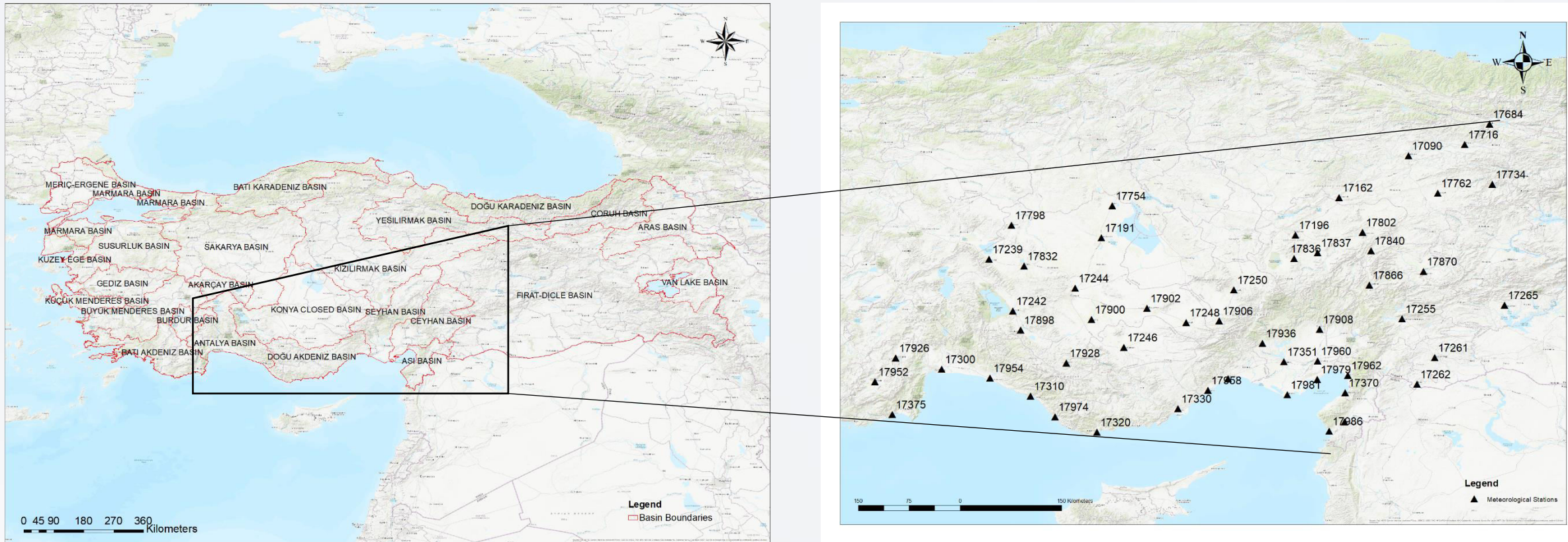


Figure 1. Locations of the MSs

- **Generalized Extreme Value (GEV) distribution for modeling extreme events**
- **Block Maxima (BM) series of the time series**
- **Three-parameter distribution: location ( $\mu$ ), scale ( $\sigma$ ) and shape ( $\xi$ )**
- **Cumulative Distribution Function:**

$$G(z) = \exp \left\{ - \left[ 1 + \xi \left( \frac{z - \mu}{\sigma} \right) \right]^{1/\xi} \right\}$$

## Four types of GEV models:

| Model | $\mu$ | $\sigma$ | $\xi$ |
|-------|-------|----------|-------|
| T0    | C     | C        | C     |
| T1    | CH    | C        | C     |
| T2    | C     | CH       | C     |
| T3    | CH    | CH       | C     |

C: Constant    CH: Changing

- **Appropriate covariates**
- **Five covariates (1, 2 and 3 based on literature, 4 & 5 suggested in this study):**
  - Tmax (maximum temperature of the day that block maxima event occurred)
  - Y (year)
  - NAO (North Atlantic Oscillation)
  - **DN (number of days in a year whose maximum temperature exceeds the long-term average temperature)**
  - **SL (a linear regression that fitted to the Tmax time series)**



## 31 GEV models

Table 1. GEV Models and Covariates

| Model Type | Model | Covariates |   |    |    |     |   |
|------------|-------|------------|---|----|----|-----|---|
|            |       | Tmax       | Y | DN | SL | NAO |   |
| T0         | 1     | -          | - | -  | -  | -   |   |
|            | T1    | 2          | + | -  | -  | -   | - |
|            |       | 3          | - | +  | -  | -   | - |
|            |       | 4          | - | -  | +  | -   | - |
|            |       | 5          | + | +  | -  | -   | - |
|            |       | 6          | - | +  | +  | -   | - |
|            |       | 7          | + | +  | +  | -   | - |
|            |       | 20         | - | -  | -  | +   | - |
|            |       | 21         | - | -  | -  | -   | + |
|            |       | 26         | - | +  | -  | -   | + |
| 27         |       | +          | - | -  | -  | +   |   |
| T2         | 8     | +          | - | -  | -  | -   |   |
|            | 9     | -          | + | -  | -  | -   |   |
|            | 10    | -          | - | +  | -  | -   |   |
|            | 11    | +          | + | -  | -  | -   |   |
|            | 12    | -          | + | +  | -  | -   |   |
|            | 13    | +          | + | +  | -  | -   |   |
|            | 22    | -          | - | -  | +  | -   |   |
|            | 23    | -          | - | -  | -  | +   |   |
|            | 28    | -          | + | -  | -  | +   |   |
|            | 29    | +          | - | -  | -  | +   |   |
| T3         | 14    | +          | - | -  | -  | -   |   |
|            | 15    | -          | + | -  | -  | -   |   |
|            | 16    | -          | - | +  | -  | -   |   |
|            | 17    | +          | + | -  | -  | -   |   |
|            | 18    | -          | + | +  | -  | -   |   |
|            | 19    | +          | + | +  | -  | -   |   |
|            | 24    | -          | - | -  | +  | -   |   |
|            | 25    | -          | - | -  | -  | +   |   |
|            | 30    | -          | + | -  | -  | +   |   |
|            | 31    | +          | - | -  | -  | +   |   |

- The best NS model for each MS
- Akaike Information Criterion (AIC)

$$AIC = 2K - \log L$$

where  $K$  is the number of independently adjusted parameters in the model and  $L$  is the maximum likelihood of the model

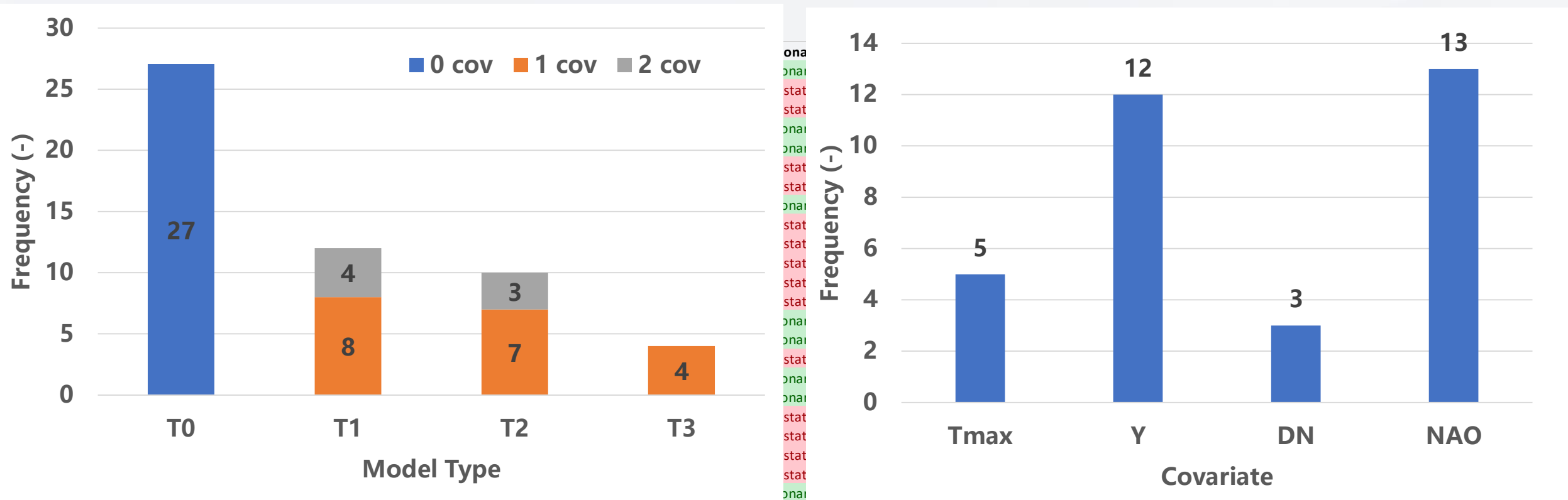
- The best NS model vs. S model
- Likelihood Ratio (LR) test

$$LR = -2(y - x)$$

where  $x$  is the negative log-likelihood of the simpler model and  $y$  is the negative log-likelihood of the complex model.

# Results

- Better performance of NS models with one or two covariates
- NS models where NAO and Y are used as covariates have better performance



|        |    |       |       |                  |                |
|--------|----|-------|-------|------------------|----------------|
| S17684 | 9  | 1.342 | 0.247 | Failed to Reject | Stationary     |
| S17701 | 24 | 6.701 | 0.034 | Rejected         | Non-stationary |
| S17734 | 21 | 2.882 | 0.090 | Failed to Reject | Stationary     |
| S17754 | 10 | 4.216 | 0.040 | Rejected         | Non-stationary |

|        |    |       |       |                  |                |
|--------|----|-------|-------|------------------|----------------|
| S17979 | 3  | 4.135 | 0.042 | Rejected         | Non-stationary |
| S17981 | 16 | 1.102 | 0.002 | Failed to Reject | Stationary     |
| S17986 | 23 | 3.708 | 0.054 | Failed to Reject | Stationary     |

Figure 2. Frequency of the Models Types

Figure 3. Frequency of the Covariates

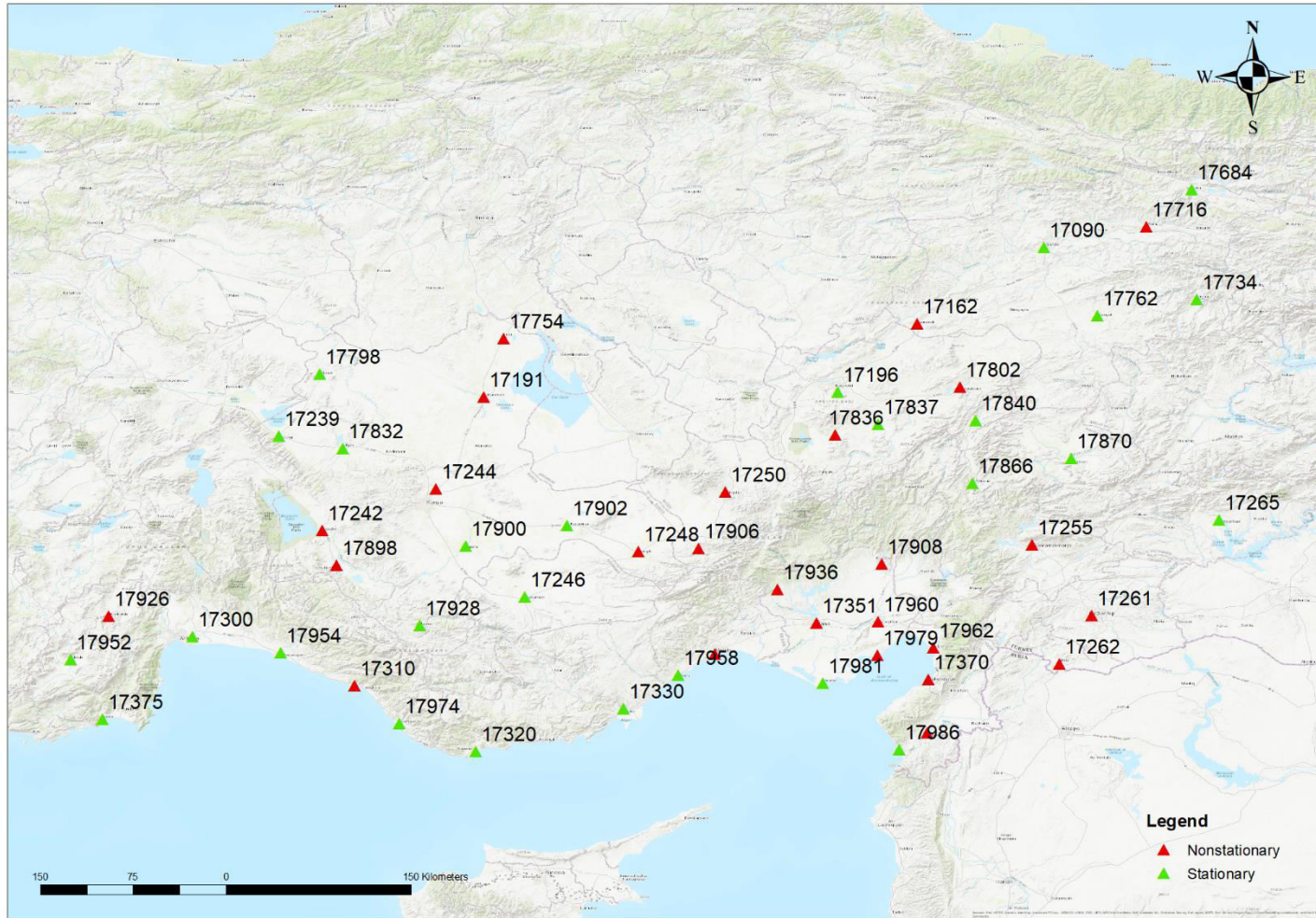


Figure 4. Stationarity of the MSs

# Conclusion

- 27 S stations & 26 NS stations
- Konya Closed Basin: 7 NS & 3 S
- Doğu Akdeniz Basin: 5 S & 1 NS
- Asi Basin: 4 NS & 1 S
- 12 T1 models, 10 T2 models and 4 T3 models
- NS models with one covariate generally performed better
- Well performing NS models generally have NAO and Y as covariates

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