



# Investigation of Heavy Rainfall Induced Runoff Analysis at Small Catchment Scale by Using Distributed Runoff Models

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# 1. Background

- Globally, about **2 billion people** are at risk of flooding, nearly **600 million** of whom live in poverty. According to the report released by the United Nations Office for Disaster Reduction (UNDRR), during the past two decades (2000-2019), a total of **3,254 flood events** have been reported worldwide, accounting for **44 %** of all disasters. A total of **1.65 billion people** were affected by floods, and about **100,000 people** were killed.



2021.7.21 Henan



2021.8.21 Shanxi



2021.8.21  
Heilongjiang



2021.10.10 Shanxi



2021.8.21 Sichuan



2021.10.8 Shanxi

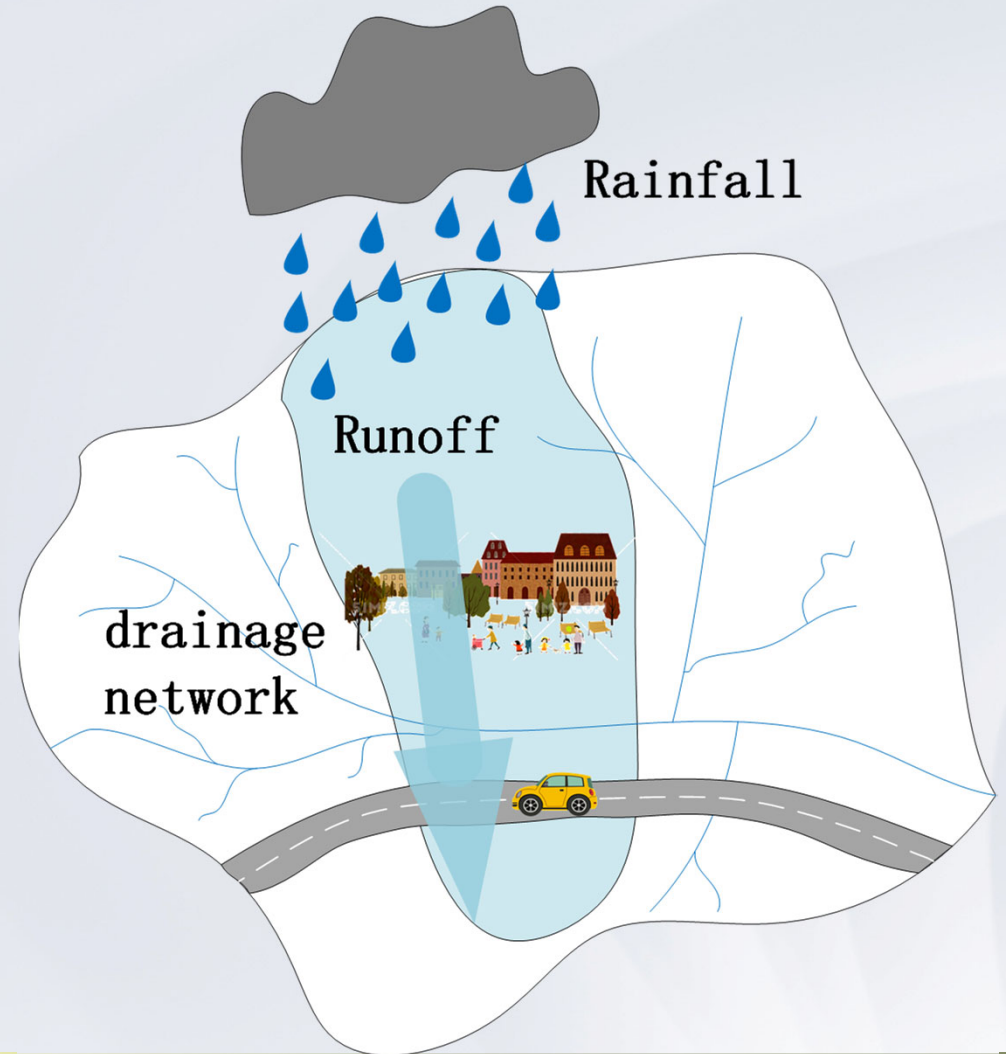




# 1. Background

- In recent years, geological disasters such as **mountain flood**, **urban flood**, and debris flows caused by heavy rainfall **have occurred frequently around the world**. The runoff process plays a vital role in the generation of these disasters and the simulation of distributed runoff depth is an important means to evaluate geological hazards.
- Therefore, many scholars are committed to solving a key problem:

**How to quickly simulate rainfall-runoff processes in mountainous or urban areas?**



## 2. Existing hydrodynamic methods

Although the equations of motion of shallow water equation can be solved as-is, the timesteps need to be small, and the equations **cannot be used for predications on the time scales of actual liquid flow problems.**

**Shallow water equations:** Equations of motion:  Diffusion wave approximation (Zhu et al., 2020):

Equation of continuity:

$$\frac{\partial(hu)}{\partial t} + \frac{\partial(hu^2)}{\partial x} + \frac{\partial(huv)}{\partial y} = G_x + F_x + D_x \quad \rightarrow \quad 0 = G_x + F_x$$

$$\frac{\partial(hv)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial(hv^2)}{\partial y} = G_y + F_y + D_y \quad \rightarrow \quad 0 = G_y + F_y$$

$$\frac{\partial h}{\partial t} + \frac{\partial(hu)}{\partial x} + \frac{\partial(hv)}{\partial y} = R - I$$

Ignore some insignificant terms to improve computing efficiency!

> 99%

Equations of motion:

**Contributions of each term in the equations of motions for surface water flows.**

$$\frac{\partial(uh)}{\partial t} + \frac{\partial(hu^2)}{\partial x} + \frac{\partial(huv)}{\partial y} = -hg \frac{\partial H}{\partial x} - hgS_{fx} + D_x$$

$$\frac{\partial(vh)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial(hv^2)}{\partial y} = -hg \frac{\partial H}{\partial y} - hgS_{fy} + D_y$$

Slope angle (°)	Inertia term (%)	Velocity term (%)	Driving force term (%)	Friction Term (%)	Advection Term (%)
	$\frac{\partial u}{\partial t}, \frac{\partial v}{\partial t}$	$\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y}, \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y}$	$G_x, G_y$	$F_x, F_y$	$D_x, D_y$
<1°	<0.001	<0.001	50.038	49.961	<0.001
5°	<0.001	0.005	49.584	50.410	<0.001
10°	<0.001	0.011	49.595	50.393	<0.001
15°	0.003	0.015	49.535	50.446	0.001
30°	0.024	0.219	52.537	47.200	0.020
45°	0.051	0.850	44.260	54.730	0.109
60°	0.159	0.327	50.808	48.679	0.027

Zhu, Y.L., Ishikawa, T., Subramanian, S. S., et al., 2020. Simultaneous analysis of slope instabilities on a small catchment-scale using coupled surface and subsurface flows. *Engineering Geology*, 275, 105750.

## 2. Existing hydrodynamic methods

### Integrated watershed model of runoff and seepage:

The governing equation of the integrated watershed model of runoff and seepage is as follows ((more details are discussed by Mori et al. (2015)).

$$-\nabla M_p - Q = \frac{\partial(\phi S_p)}{\partial t}, \quad p = (\text{water, air})$$

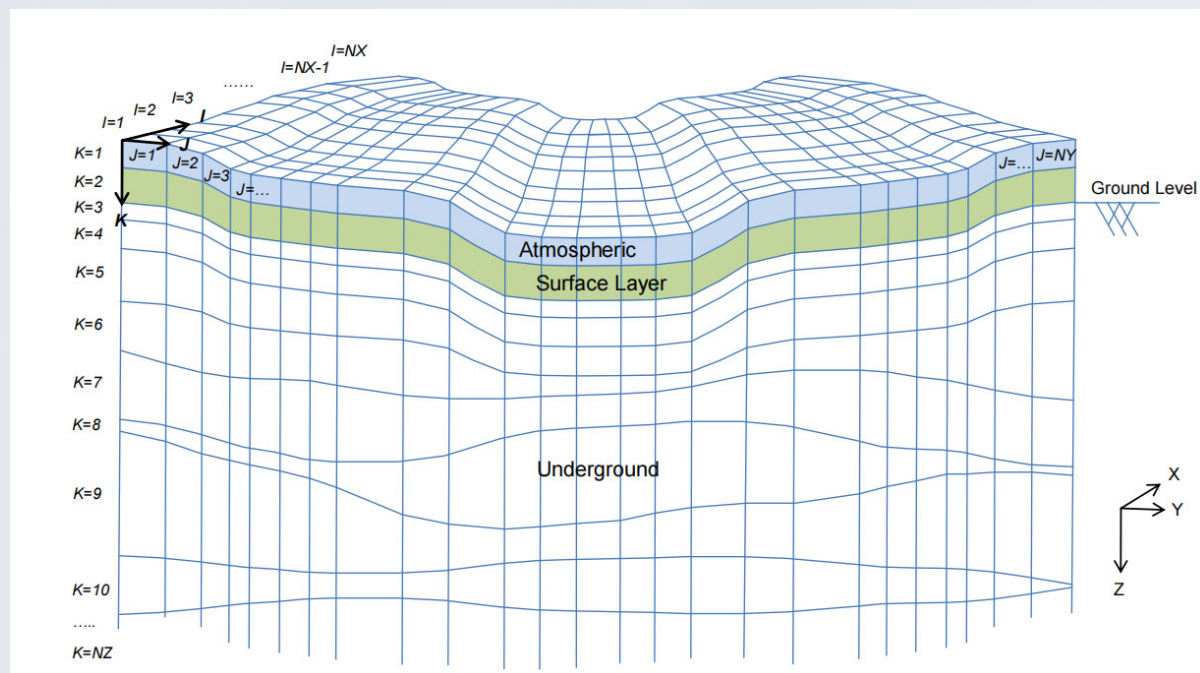
Runoff:

$$M_w = -\frac{h^{2/3}}{n_m \sqrt{|S|}} \nabla(h + z)$$

$$M_a = -\frac{k_s k_{r,a}}{\rho_a g} \cdot \nabla(\psi_a)$$

Seepage:

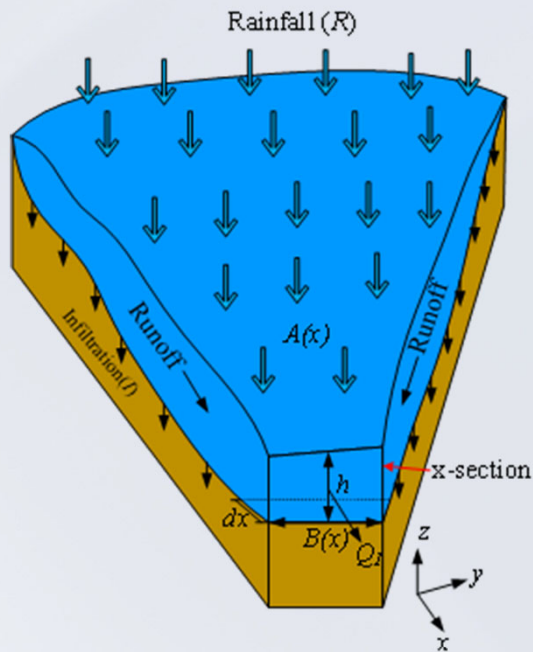
$$M_p = -\frac{k_s k_{r,p}}{\rho_p g} \cdot \nabla(\psi_p)$$



Mori, K., Tada, K., Tawara, Y., et al., 2015. Integrated watershed modeling for simulation of spatiotemporal redistribution of post-fallout radionuclides: application in radiocesium fate and transport processes derived from the Fukushima accidents. *Environmental Modelling & Software*, 72, 126-146.

# 3. Distributed Runoff Model (DRM)

Distributed Runoff Model (DRM)----A fully integrated hydrological-hydrodynamic model:



$$\left. \begin{aligned} \frac{\partial h}{\partial t} + \frac{\partial(Q_1)}{\partial l} &= R - I \\ Q_1 &= ah^m \\ a &= \frac{\sqrt{s}}{n_m} \end{aligned} \right\}$$

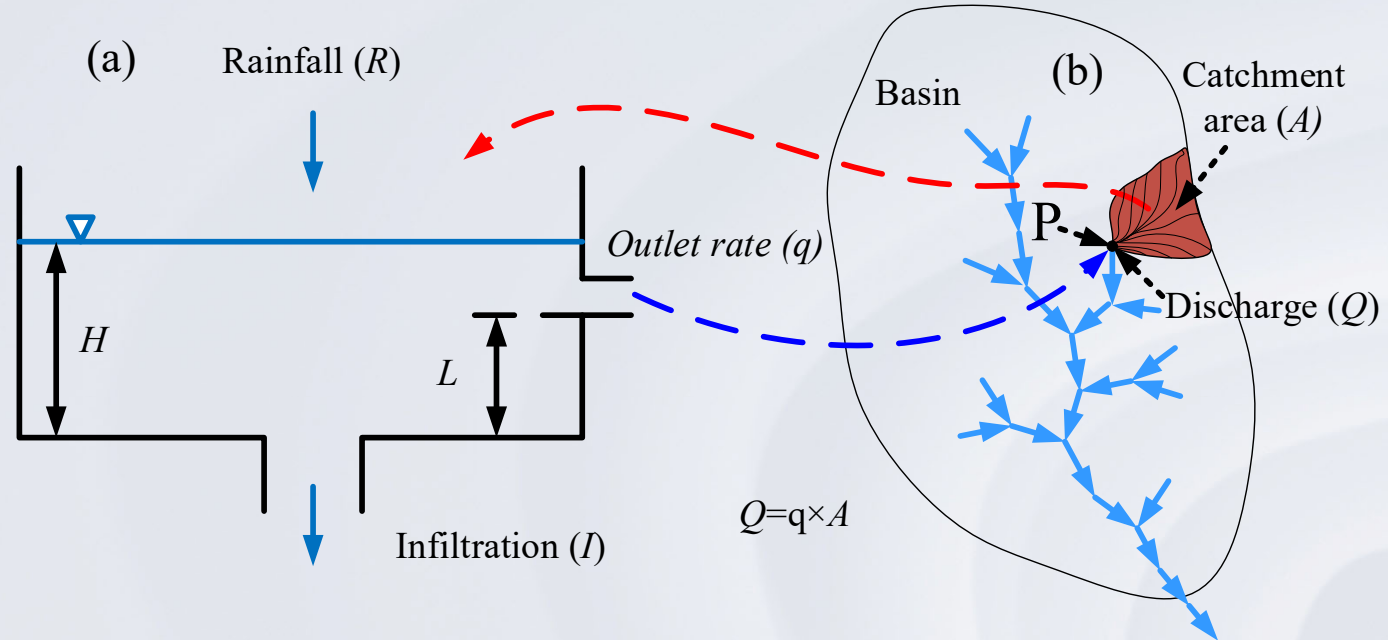
Hydrodynamic model

Establish a mathematical association



Hydrological model

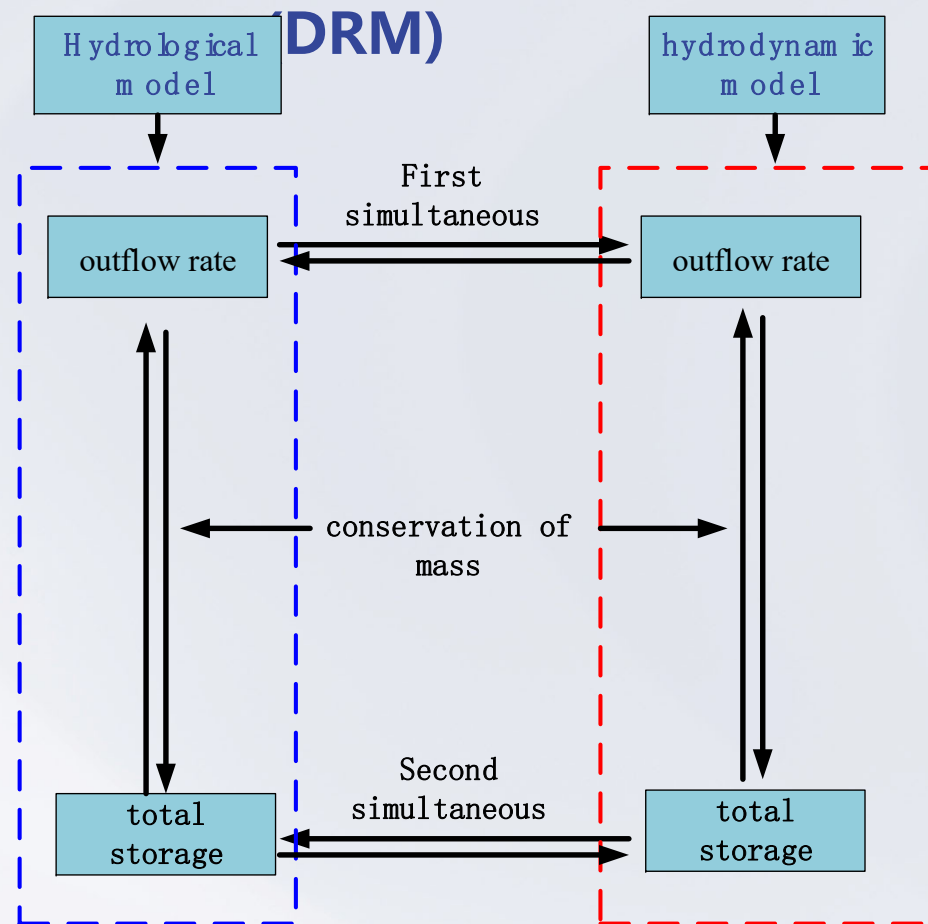
$$\left[ \begin{aligned} \frac{dH}{dt} &= R - I - q \\ Q &= q \times A \end{aligned} \right.$$





# 3. Distributed Runoff Model (DRM)

## Distributed Runoff Model



Establish a mathematical association

$$\left. \begin{aligned} \frac{\partial h}{\partial t} + \frac{\partial(Q_1)}{\partial l} &= R - I \\ Q_1 &= ah^m \\ a &= \frac{\sqrt{S}}{n_m} \end{aligned} \right\} \longleftrightarrow \left\{ \begin{aligned} \frac{dH}{dt} &= R - I - q \\ Q &= q \times A \end{aligned} \right.$$

fully integrated hydrological-hydrodynamic model (Zhu et al., 2022)

$$\begin{cases} \frac{dH_x}{dt} = R - I - q_x \\ H_x = \left(\frac{n_m}{\sqrt{S}}\right)^{0.6} q_x^{0.6} b A(x)^c \\ h(x) = \left(\frac{n_m}{\sqrt{S}}\right)^{0.6} (k q_x)^{0.6} \left(\frac{A(x)}{B(x)}\right)^{0.6} \\ V(x) = \frac{\sqrt{s_p}}{n_m} h(x)^{\frac{2}{3}} \end{cases}$$

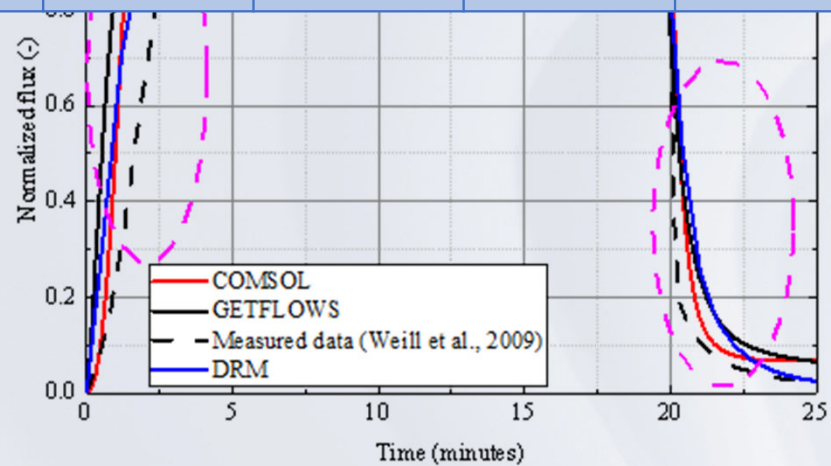
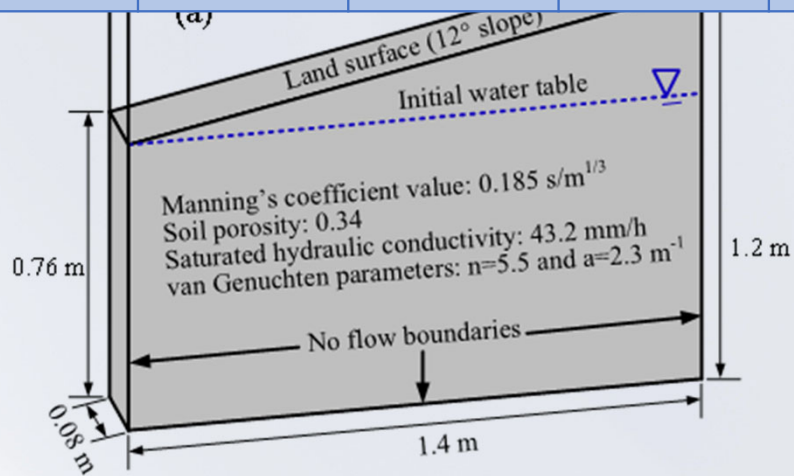
Zhu, Y.L., Zhang, Y.F., Yang, J., Nguyen, B. T., & Wang, Y.\* (2022). A novel method for calculating distributed water depth and flow velocity of stormwater runoff during the heavy rainfall events. Journal of Hydrology, 612, 128064.



# 4. Effectiveness of DRM

## Validation of DRM by Abdul and Gillham

Parameter	$n_m$ (s/m <sup>1/3</sup> )	S (1)	b	c	k	L (mm)	A(x) (km <sup>2</sup> )	B(x) (m)	f (1/h)
Value	0.185	0.105	3	0.21	1	0	$0.112 \times 10^{-6}$	0.08	0

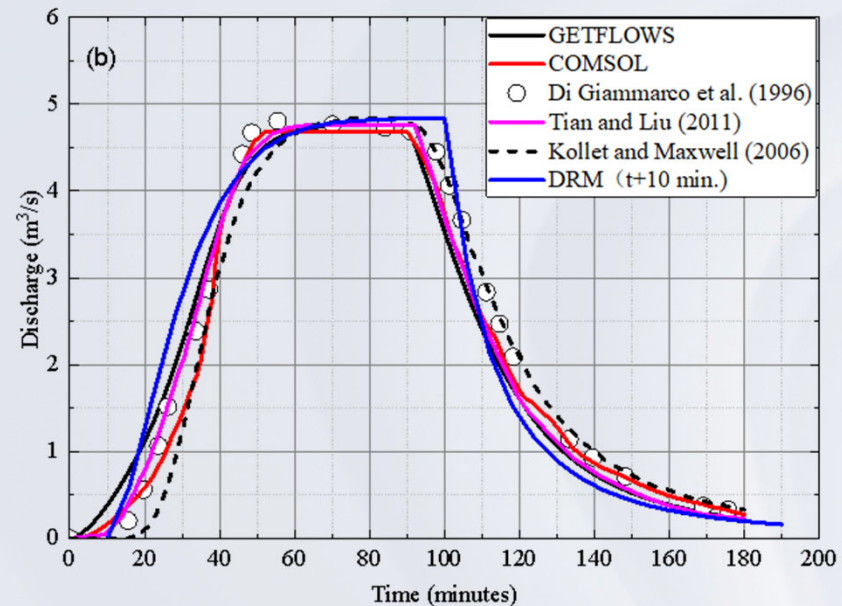
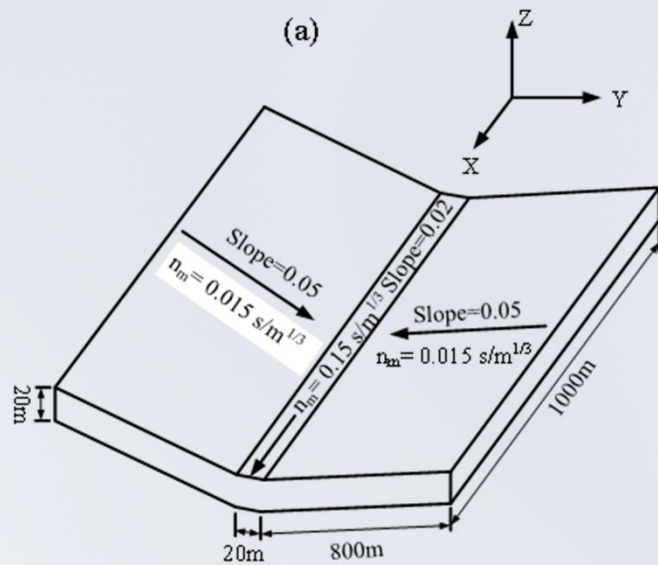


Accordingly, by using the Abdul and Gillham system, the effectiveness of DRM on the scale of the test chamber (the catchment area is  $0.112 \times 10^{-6} \text{ km}^2$ ) has been well verified.

# 4. Effectiveness of DRM

## Validation of DRM by V-catchment system

Parameters	$n_m$ (s/m <sup>1/3</sup> )	$S$ (1)	$b$	$c$	$k$	$L$ (mm)	$A(x)$ (km <sup>2</sup> )	$B(x)$ (m)	$f$ (1/h)
Value	0.015	0.05	4	0.21	1	0	1.62	20	0

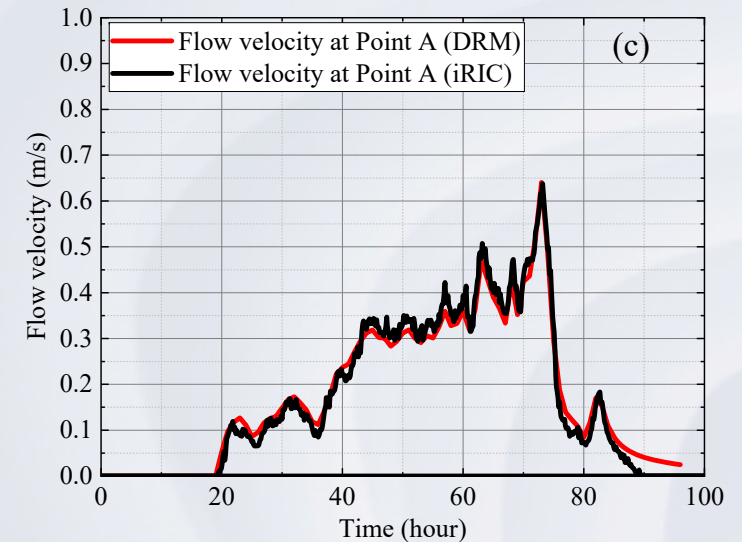
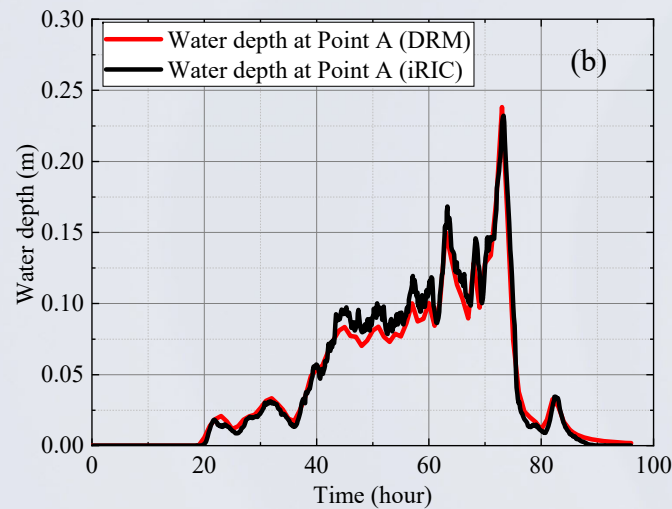
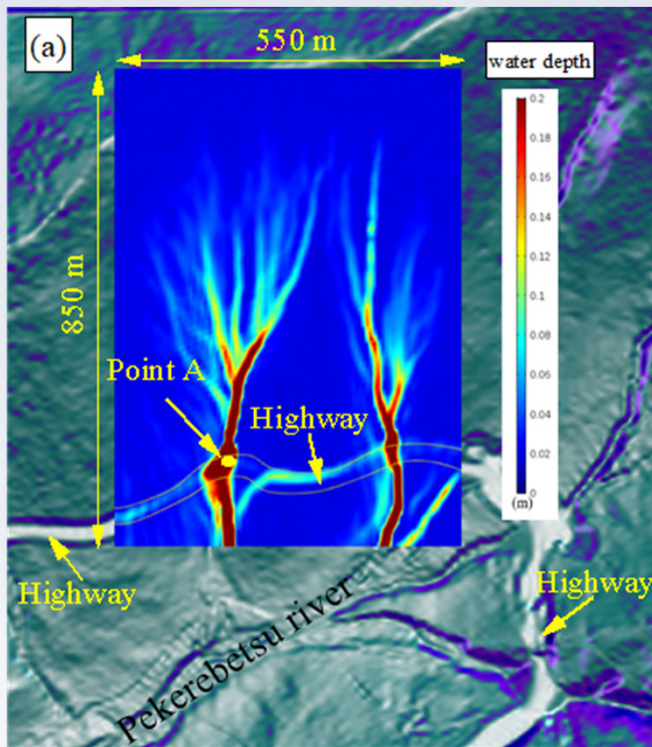


Accordingly, by using the Abdul and Gillham system, the effectiveness of DRM on the scale of the test chamber (the catchment area is  $0.112 \times 10^{-6}$  km<sup>2</sup>) has been well verified.

# 4. Effectiveness of DRM

## Validation of DRM in a real catchment

Parameters	$n_m$ (s/m <sup>1/3</sup> )	$S$ (1)	$S_p$ (1)	$b$	$c$	$k$	$L$ (mm)	$A(x)$ (km <sup>2</sup> )	$B(x)$ (m)
Value	0.3	0.25	0.1	4	0.21	1	0	0.1356	20



The water depth and flow velocity calculated by DRM are almost completely identical with the simulation results of the iRIC (shallow water equations) at different times, which shows that DRM still has a very high calculation accuracy on the real catchment scale.

## 4. Effectiveness of DRM

### The improvement of calculation efficiency

Model	Catchment area	Simulation time	Calculation time of shallow water equations	Calculation time of DRM	The improvement of calculation efficiency
Test Model	$1 \times 10^{-3}$	1 hour	14 s	4 s	<b>71.4%</b>
		1 day	1 min. 33 s	27 s	<b>71.0%</b>
		3 day	6 min. 12 s	84 s	<b>77.4%</b>
Catchment Model	0.1356	3 hour	14 min 57 s	One Point: 4 s	<b>99.5%</b>
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The calculation efficiency of DRM has increased by 78.6%, 87.1%, and 89.0% respectively, so the calculation efficiency has increased by 70% ~ 90%.



## 5. Conclusions

- A fully integrated hydrological-hydrodynamic model-DRM is proposed to calculate the distributed water depth and flow velocity of heavy rainfall-induced stormwater runoff under time-dependent rainfall by fully coupling the conceptual hydrological model and hydrodynamic model.
- The effectiveness and practicality of DRM on different catchment scales (i.e.,  $0.112 \times 10^{-6} \text{ km}^2$ ,  $1 \times 10^{-3} \text{ km}^2$ , and  $1.62 \text{ km}^2$ ) have been verified by comparing with the test results and three other methods, i.e., shallow water equations, integrated watershed model, and iterative cross-coupled model of runoff and seepage.
- Results indicate that the DRM proposed in this study greatly improves the calculation efficiency and reduces the calculation difficulty of the distributed water depth and flow velocity of the heavy rainfall-induced stormwater runoff while meeting the accuracy requirements.

**Thank you very much for your kind  
attention and comments**