Potential Evaporation and the Complementary Relationship

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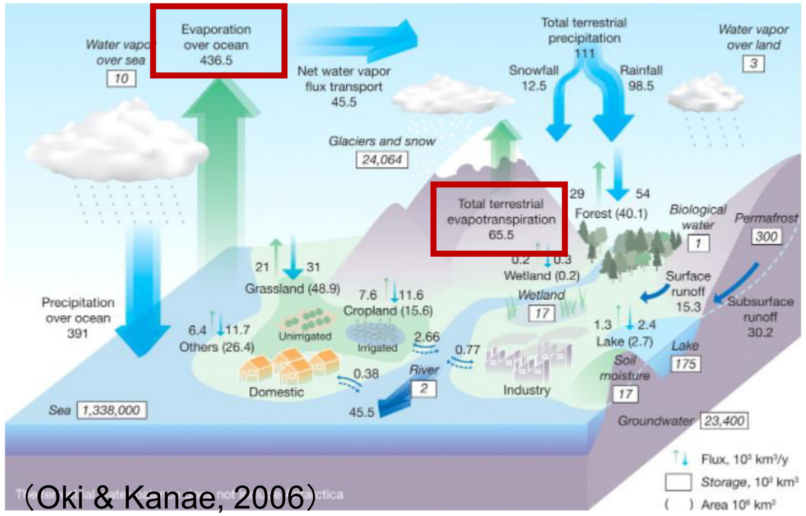
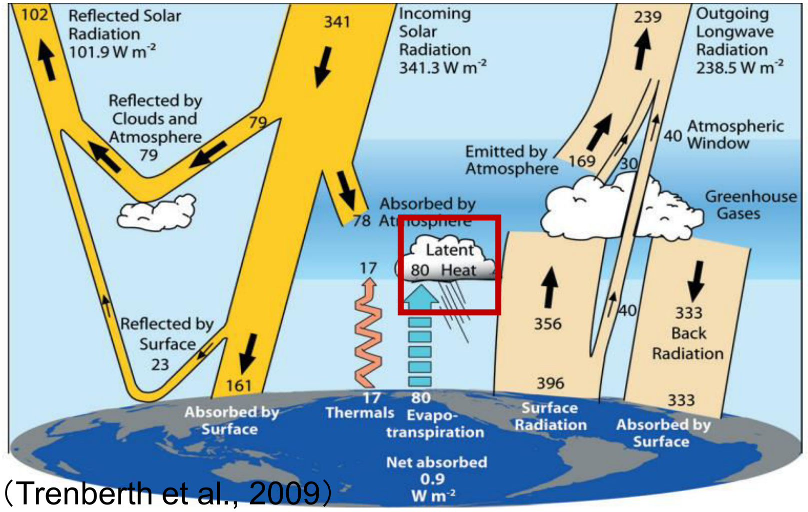
Evaporation (*E*) is a crucial linkage between water and energy cycles

（Oki & Kanae, 2006; Trenberth et al., 2009）



（Oki & Kanae, 2006）



（Trenberth et al., 2009）

Water Budget $P = E + R + \Delta S$

The 2nd largest water flux in the terrestrial water cycle

Energy Budget $R_n = LE + H + G$

Consuming 82% of the energy gained on the Earth's surface

Potential Evaporation is a key concept to estimate *E*

- Actual Evaporation ：

  The evaporation under natural conditions

- **Potential Evaporation ：**

  The evaporation occurs with unlimited water supply

Models for estimating the potential evaporation

Combination method（Penman, 1948）

$$LE = \frac{\Delta R_n + \rho_a C_p \frac{D_a}{r_a}}{\Delta + \gamma}$$

areodynamic term + radiative term

Coupling the aerodynamic and radiative term, it is considered to be a more reliable model.

Energy-based method（Priestley & Taylor, 1972）

$$LE = \alpha \frac{\Delta}{\Delta + \gamma} R_n$$

equilibrium evaporation

minimal advection

minimal areodynamic term + radiative term

Simplifying aerodynamic term with minimum advection,

it has good performance in E_P estimation

■ Problems of classic potential evaporation models

- **Potential evaporation:** The evaporation that would occur **when the surface is saturated**

Priestley-Taylor $LE = \alpha \frac{\Delta}{\Delta + \gamma} R_n$

Penman $LE = \frac{\Delta R_n + \rho_a C_p \frac{D_a}{r_a}}{\Delta + \gamma}$

Meteorological forcing variables observed in real conditions do not necessarily represent that measured over hypothetical wet surfaces

?

Temperature	<	Temperature
Solar radiation	≈	Solar radiation
Longwave radiation	>	Longwave radiation
Humidity	>	Humidity

■ Problems of classic potential evaporation models (Brutsaert, 2015)

Potential Evaporation (E_{po})

(large) surface is wet and the surrounding air is humid

Apparent Potential Evaporation (E_{pa})

(small) surface is wet but the surrounding air is dry

?

Temperature < **Temperature**

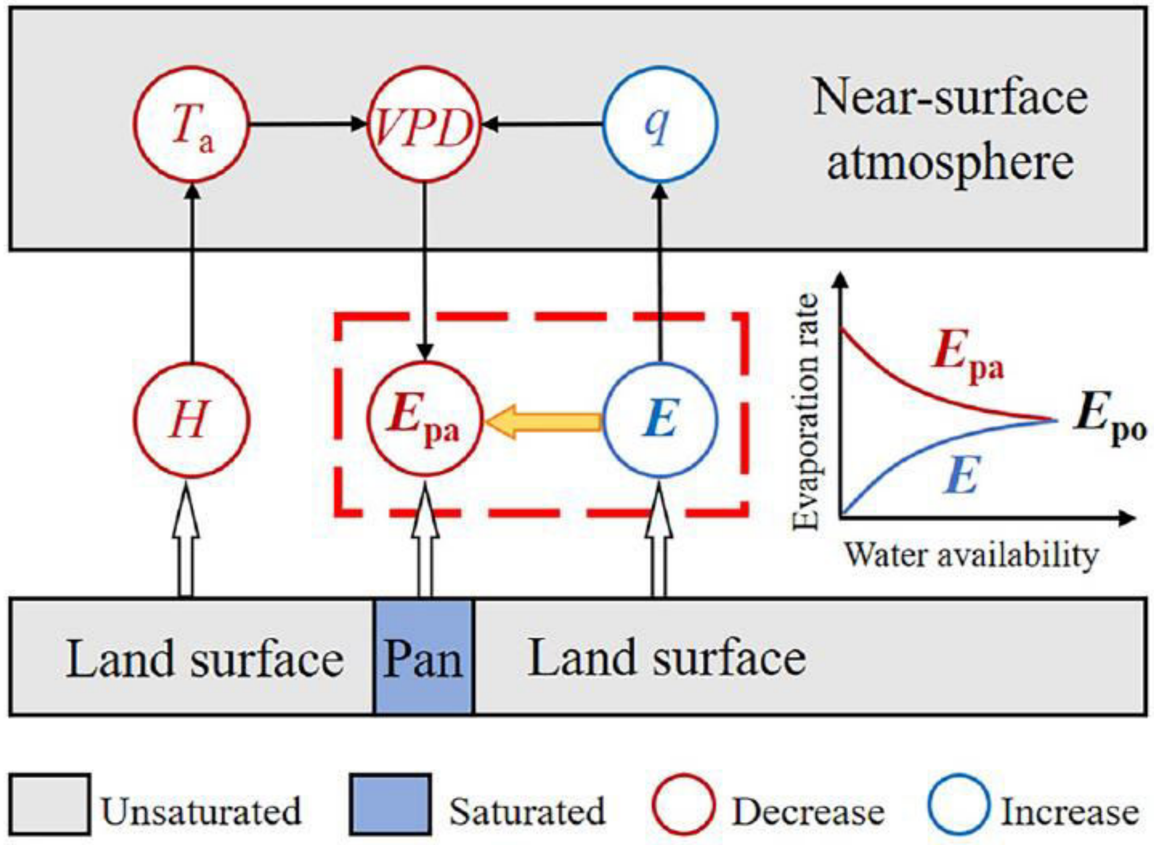
Solar radiation ≈ **Solar radiation**

Longwave radiation > **Longwave radiation**

Humidity > **Humidity**

The Complementary

(Bouchet, 1963)



The CR provides a framework for estimating *E* with basic meteorological observations based on:

$$E = f(E_{pa}, E_{po})$$

Apparent Potential Evaporation (E_{pa}):

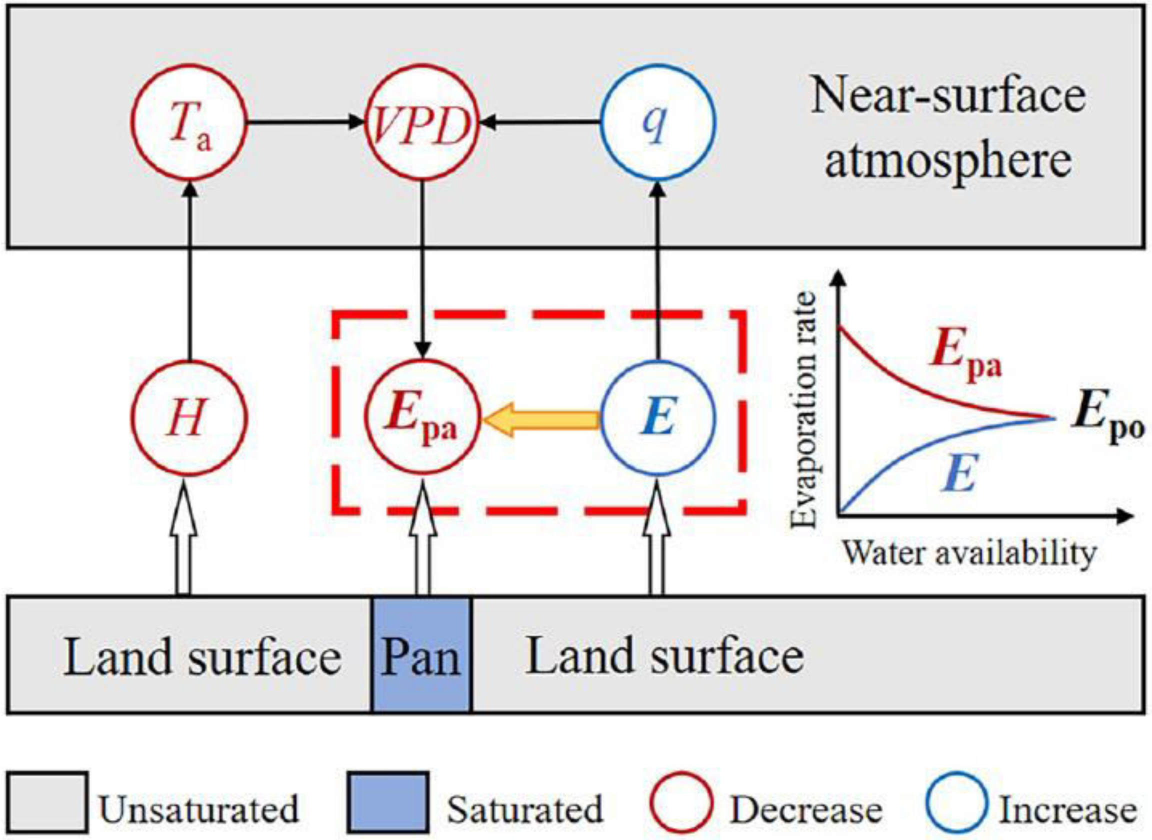
(small) surface is wet but the surrounding air is dry

Potential Evaporation (E_{po}):

(large) surface is wet and the surrounding air is humid

The Complementary

(Bouchet, 1963)



Apparent Potential Evaporation (E_{pa}):

(small) surface is wet but the surrounding air is dry

Pan observation/ Penman equation

Debated!

Potential Evaporation (E_{po}):

(large) surface is wet and the surrounding air is humid

Meteorological variables observed in real conditions do not necessarily represent that measured over wet surfaces

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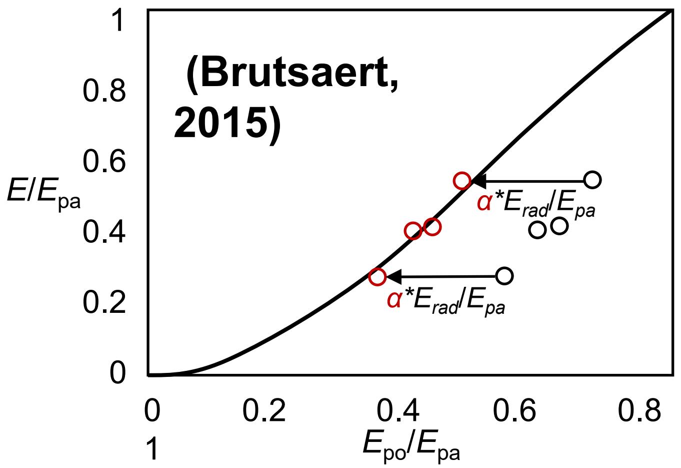
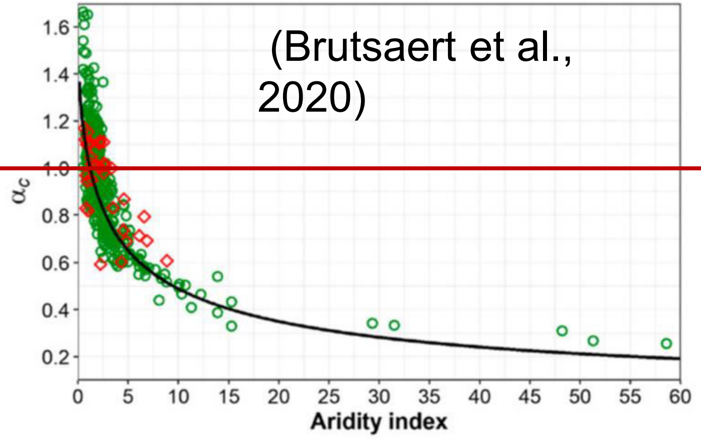
Problems of E_{po} estimation in the CR

- **Polynomial CR (Brutsaert, 2015)**

$$\frac{E}{E_{pa}} = 2\left(\frac{\alpha E_{rad}}{E_{pa}}\right)^2 - \left(\frac{\alpha E_{rad}}{E_{pa}}\right)^3$$

$$E_{po} = \alpha \frac{\Delta}{\Delta + \gamma}(R_n - G) = \alpha * E_{rad}$$





Problem: (Phillip, 1987)

Fitted α lower than 1

α should be higher

It has than 1 interna

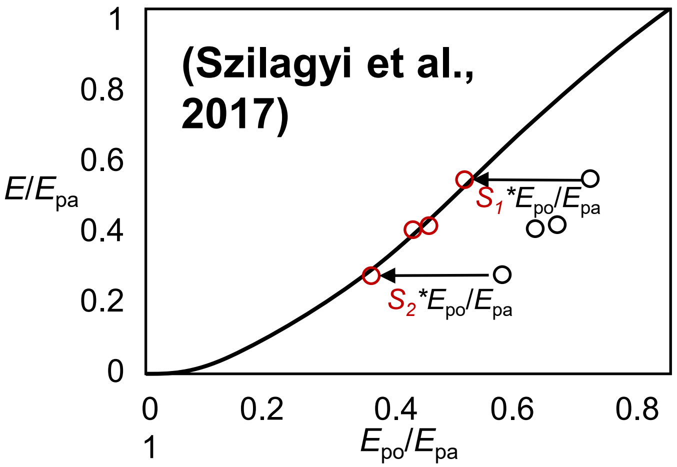
inconsistency!

Problems of E_{po} estimation in the CR

- **Rescaled CR (Szilagyi et al., 2017)**

$$\frac{E}{E_{pa}} = 2\left(S\frac{E_{po}}{E_{pa}}\right)^2 - \left(S\frac{E_{po}}{E_{pa}}\right)^3$$

$$S = \frac{E_{pa\_max} - E_{pa}}{E_{pa\_max} - E_{po}}$$



$$E_{po} = \alpha \frac{\Delta(T_{ws})}{\Delta(T_{ws}) + \gamma}(R_n - G)$$

Derive the wet surface temperature

Assume a constant R_n

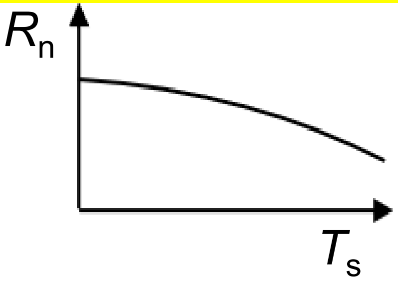
$$\beta = \frac{H}{LE} \approx \frac{R_n - G - LE_{P\_PO}}{LE_{P\_PO}} \approx \gamma \frac{T_{ws} - T_a}{e_s(T_{ws}) - e(T_a)}$$

Problem:

Ignore the interdependence between R_n and T_s

Energy Budget : $R_n = R_{si} - R_{so} + R_{li} - R_{lo}$

Stefan-Bolzmann law : $R_{lo} = (1+\varepsilon) * R_{li} + \varepsilon\sigma T_s^4$



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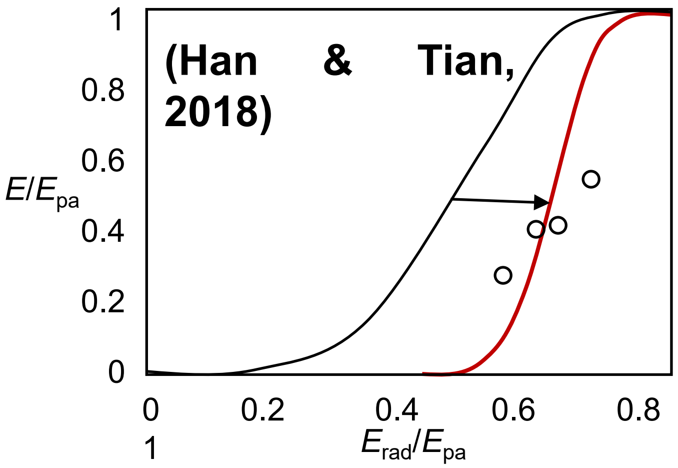
Problems of E_{po} estimation in the CR

- **Sigmoid CR (Han & Tian, 2018)**

$$\frac{E}{E_{Pen}} = \frac{1}{1+m(\frac{E_{Pen}}{E_{rad}}-1)^n}$$



Problem:

- **Abandon the concept of Epa and Epo and has a vague physical interpretation**
- **Require calibration and prevents application over regions without prior knowledge**

Problems of E_{po} estimation in the CR

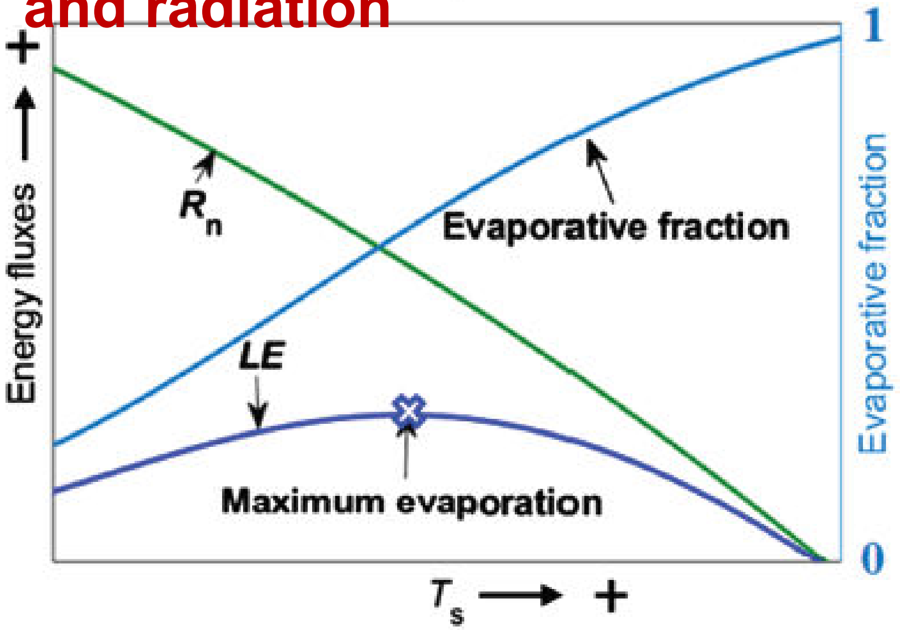
Temperature	<	Temperature
Solar radiation	≈	Solar radiation
Longwave radiation	>	Longwave radiation
Humidity	>	Humidity
Wind speed	≈	Wind speed

Key points:

- Unreasonable estimations of E_{po} hinder the development of CR
- How to estimate E_{po} following the definition (restore relevant variables to wet conditions) ?

The maximum evaporation theory（Yang & Roderick, 2019)

Acknowledging the inter-dependence between evaporation, surface temperature and radiation



Energy balance: $R_n – G = H + LE$

Bowen ratio: $\beta = H / LE$

E_P formulation: $LE = (R_n – G) * 1 / (1 + \beta)$

$T_s \uparrow$ $R_n \downarrow$ $1/(1+\beta) \uparrow$ LE_{max}

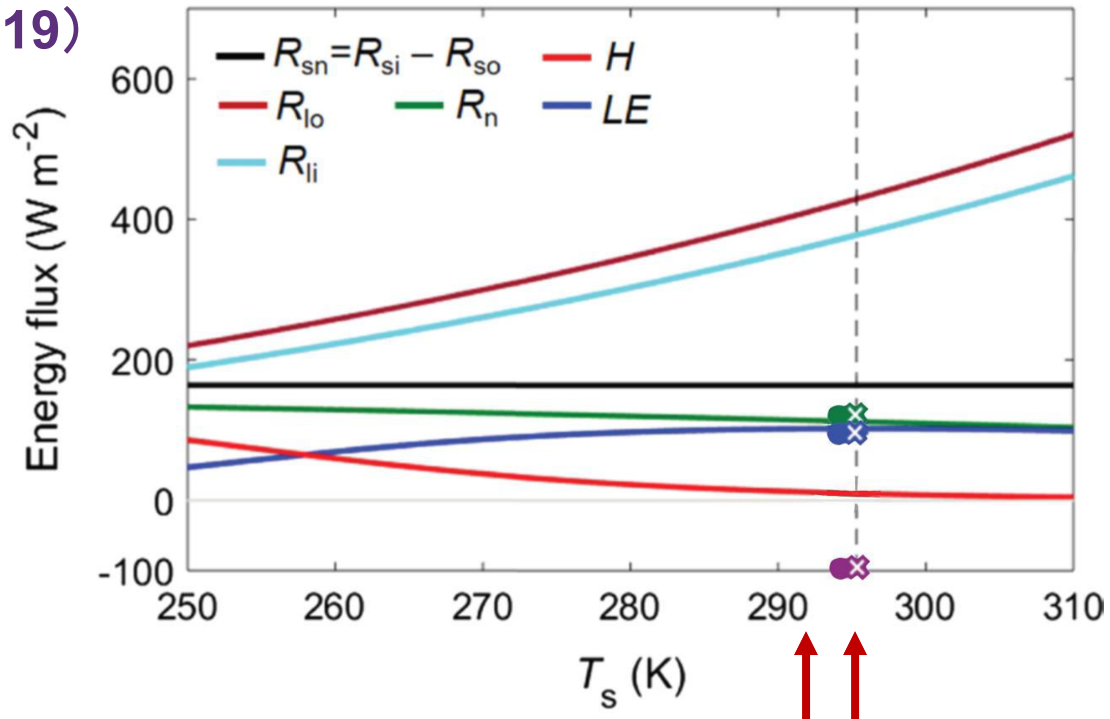
LE_{max} naturally emerges from the coupling of R_n-T_s-E

$$LE=\frac{R_{si}-R_{so}+\varepsilon\sigma(T_s-\Delta T)^4-\varepsilon\sigma T_s^4}{1+\beta(T_s)}=\frac{R_n(T_s)}{1+\beta(T_s)}$$

Energy balance : $R_n=R_{si}–R_{so}+R_{li}–R_{lo}$

Stefan-Bolzmann law : $R_{lo}=(1+\varepsilon)*R_{li}+\varepsilon\sigma T_s^4$

■ The maximum evaporation theory（Yang & Roderick, 2019）



$$LE=\frac{R_{si}-R_{so}+\varepsilon\sigma(T_s-\Delta T)^4-\varepsilon\sigma T_s^{\,4}}{1+\beta(T_s)}=\frac{R_n(T_s)}{1+\beta(T_s)}$$

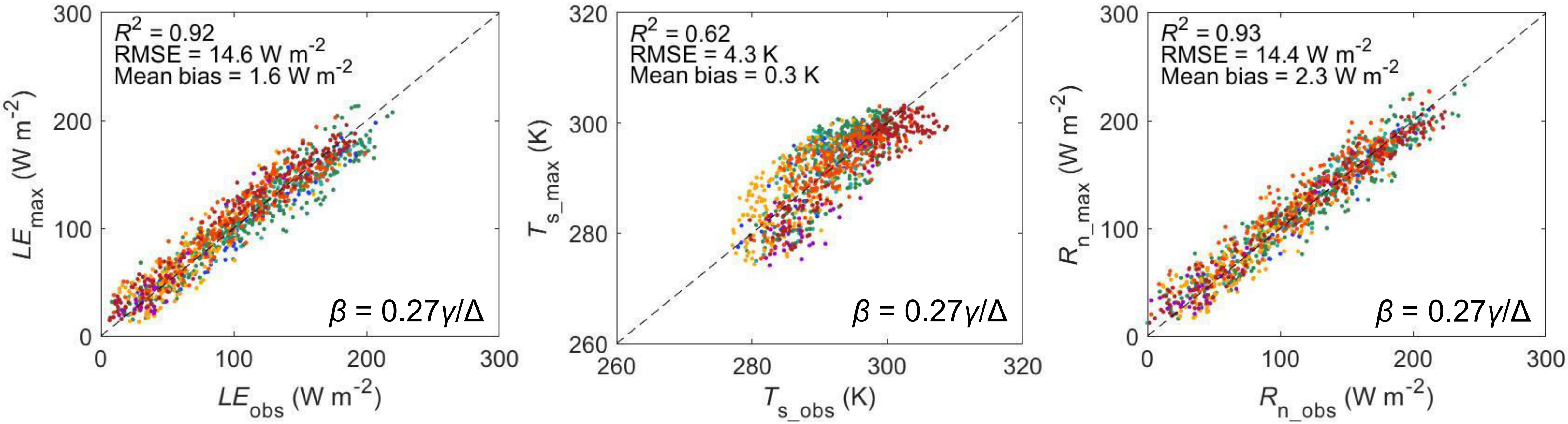
- **This theory can restore *E*, R_n and T_s to water surface conditions.**
- **This theory has the potential to estimate E_P over land surfaces!**

Observed *LE*	102.7 W m^{-2}
Observed T_s	293.8 K
Observed R_n	113.7 W m^{-2}

Max. *LE*	102.3 W m^{-2}
T_s at Max. *LE*	295.3 K
R_n at Max. *LE*	112.5 W m^{-2}

Consistent !

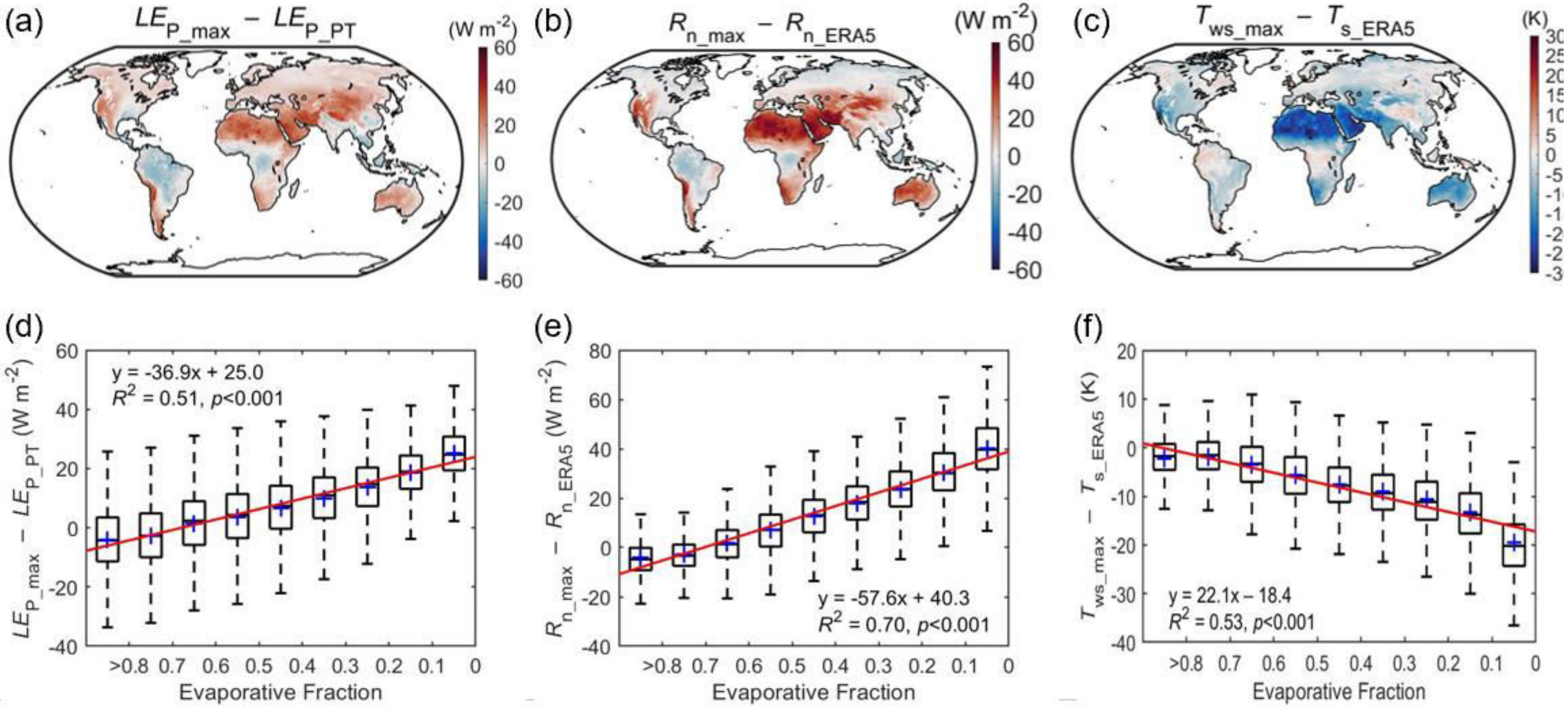
Validation of the maximum evaporation method over land surface (Tu et al., 2022; HESS)



The maximum evaporation method can restore E, T_s and R_n to wet conditions over land surface.

■ E_P estimates under both wet and dry conditions

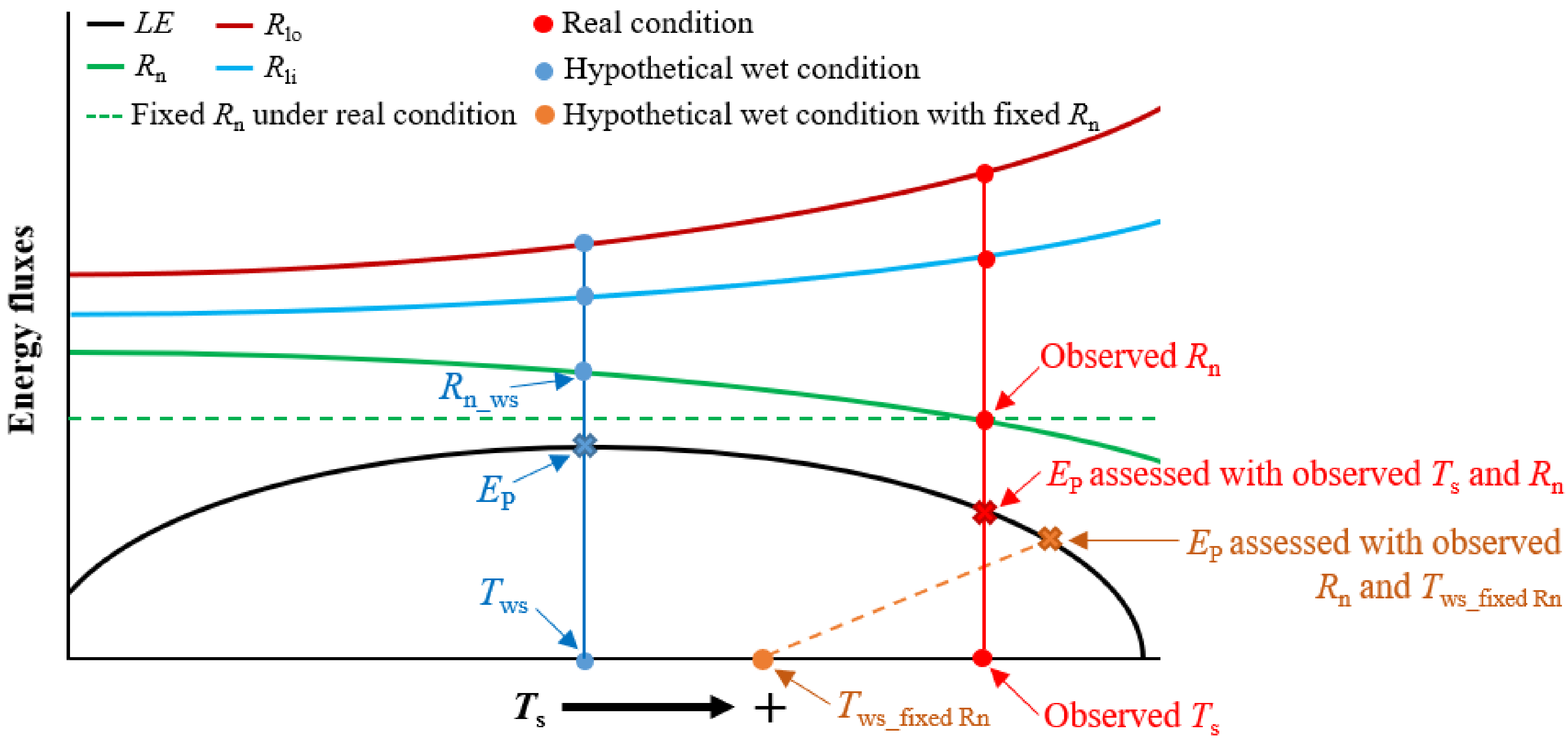
(Tu & Yang, 2022; WRR)



The underestimation of E_{P_PT} is caused the observed higher T_a and lower R_n under conditions.

E_P estimates under both wet and dry conditions

(Tu & Yang, 2022; WRR)



It is crucial to restore all the relevant variables (T_s, R_n) to wet conditions in estimating E_P !

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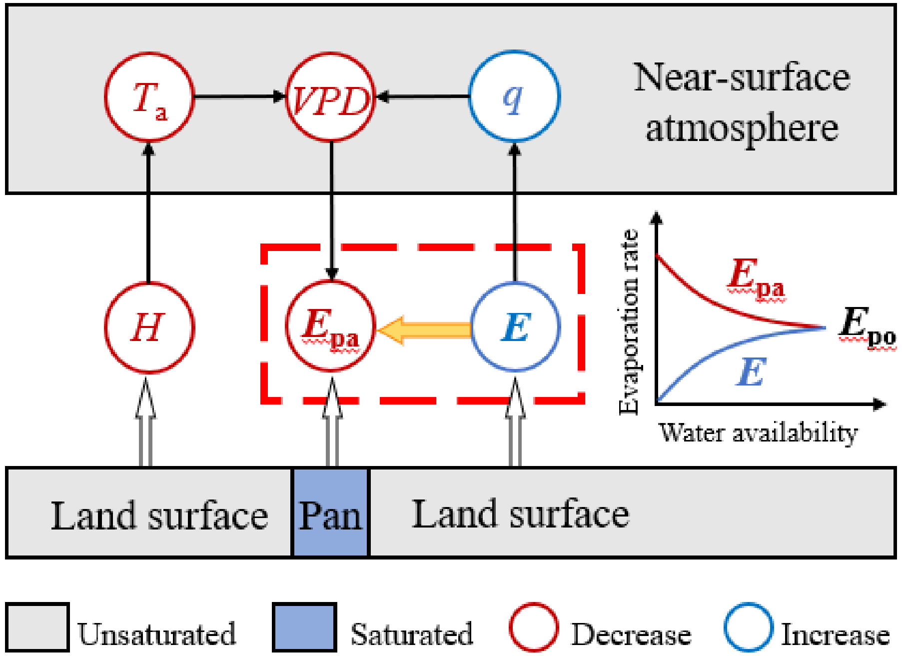
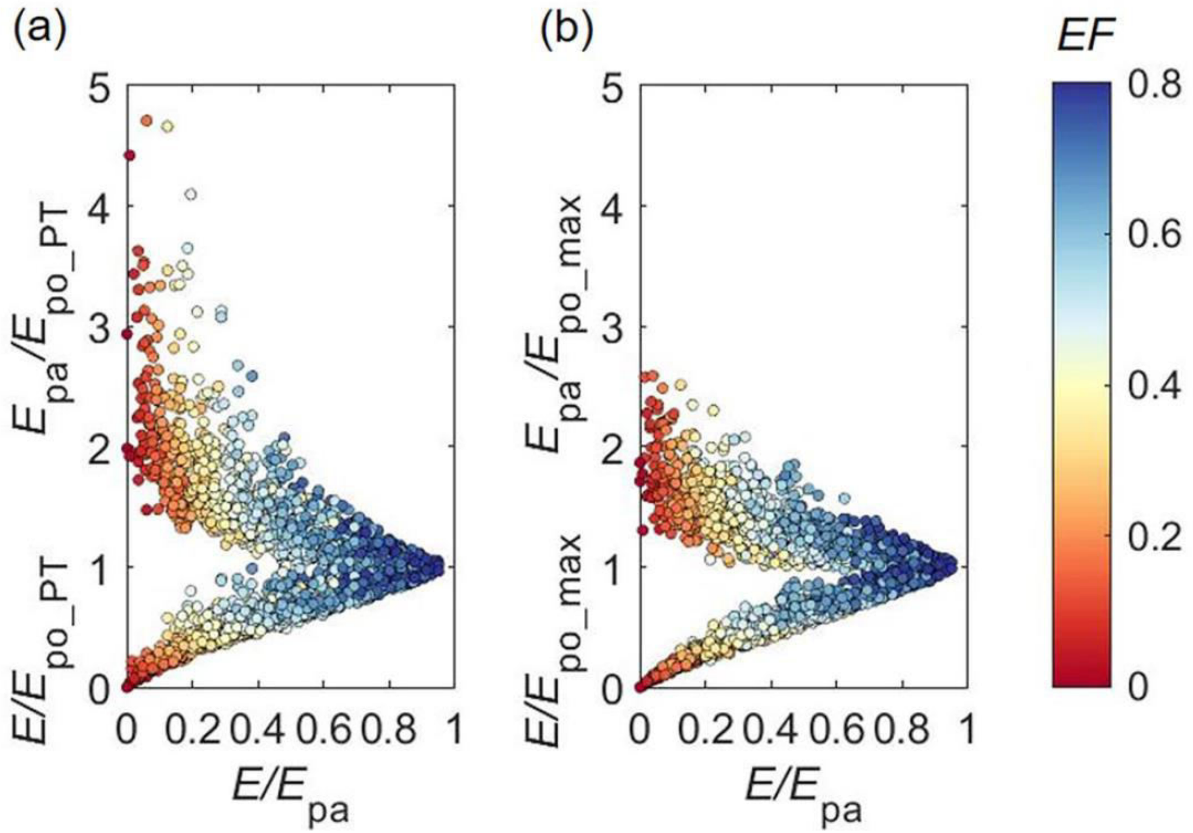
■ CR with different E_{po} calculations

(Tu et al., 2023; WRR)

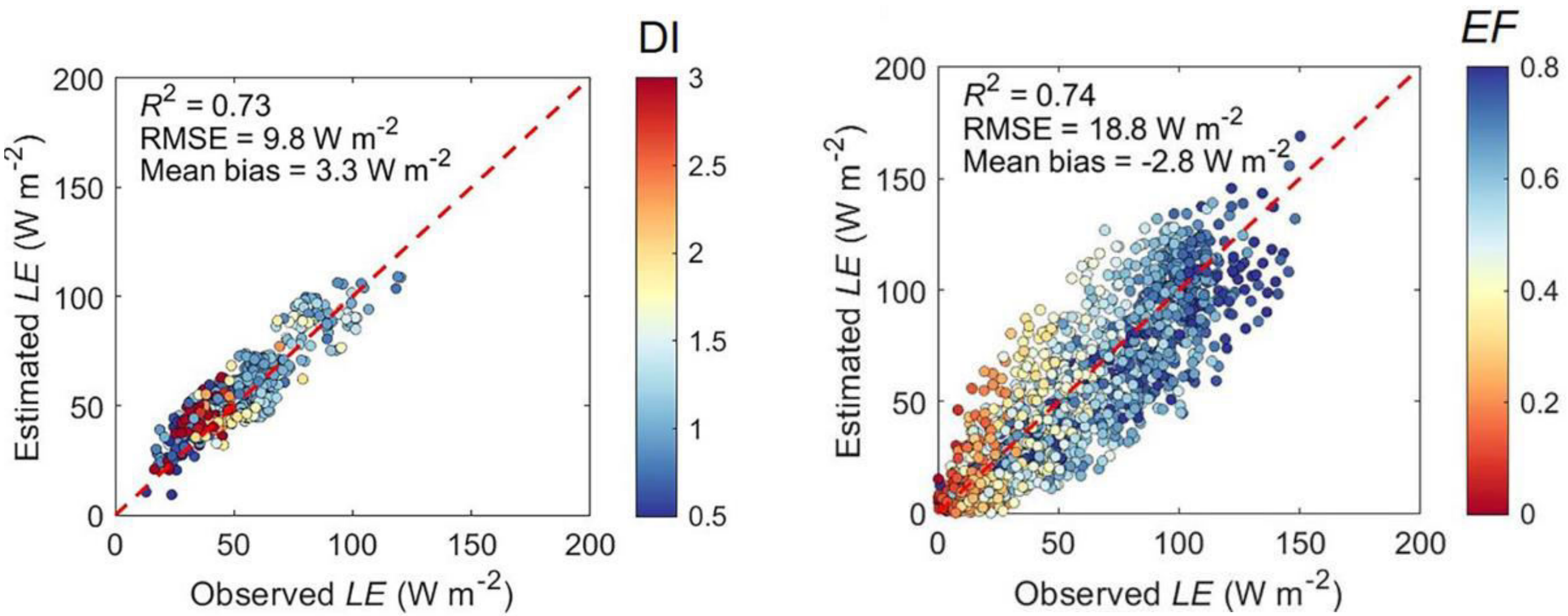




Adopting E_{po_max} considerably reduces the asymmetry of the complementarity in CR.

■ New CR model with E_{po_max}

(Tu et al., 2023; WRR)



A physically based CR model with E_{po_max} was validated to have good performance in *E* estimation.

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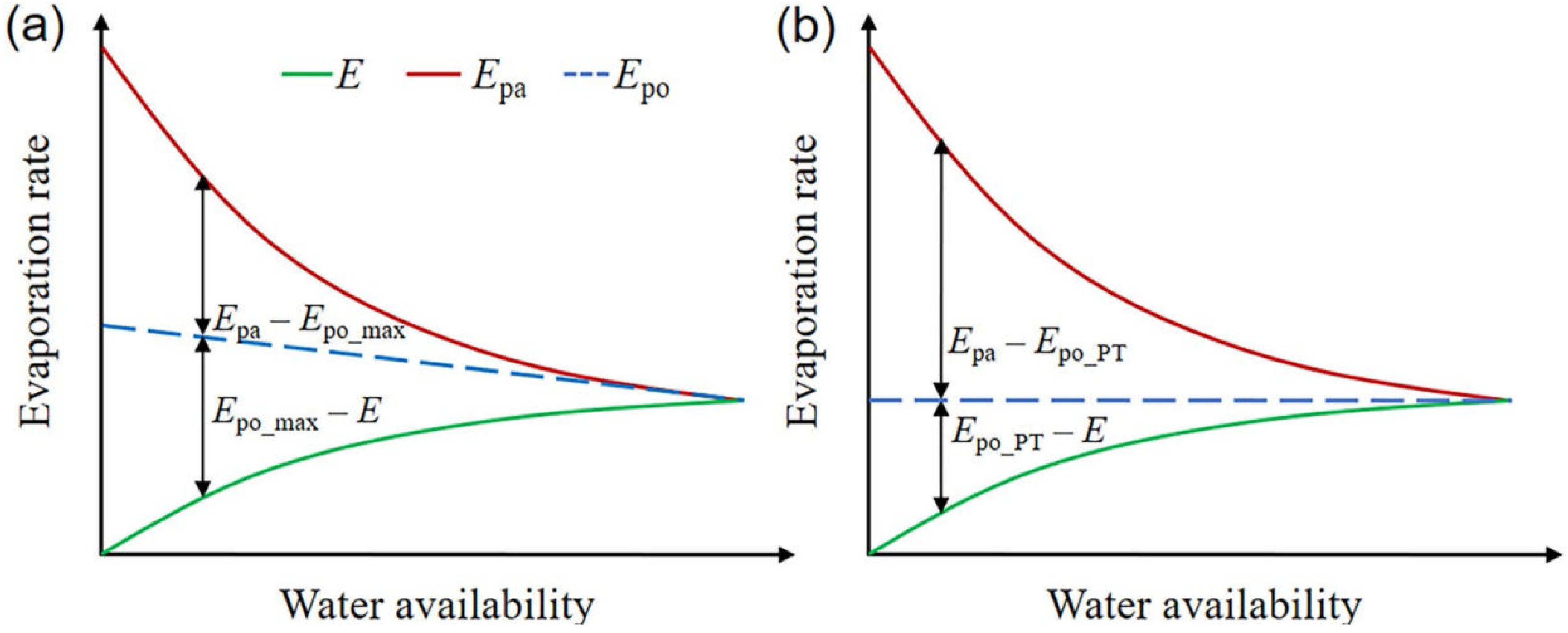
■ Take-home messages



- The maximum evaporation method can **recover the variables to a hypothetical wet condition**
- Adopting E_{po_max} that conforms to the physical definition **reduces the asymmetry**

Thanks for your attention!

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