ANALYSIS ACROSS TIME SCALES OF RAINFALL EXTREMES: SCALING AND NON-SCALING APPROACHES

M. Isabel P. de Lima

Marine and Environmental Research Centre, Institute of Marine Research; ESAC/Polytechnic Institute of Coimbra, Portugal (iplima@esac.pt)

Abstract

Probable impacts of extreme precipitation on society and the environment are strongly determined by regional specificities and are expected to be affected by changes in the climate. Thus, the study of local precipitation regimes is essential in this context. However, many models have difficulties in adequately describing the large variability in precipitation, which is still a task that is in need of further insight. This work reports analyses of rainfall extremes across time scales, which is conducted in a multifractal framework that handles large rainfall dynamic ranges, brings together the behavior of rainfall observed at different scales and overcomes the problem of using different models to describe data of different resolutions. Moreover, this framework implicitly assumes the presence of heavy tails in the probability distribution of rainfall intensity. Results suggest that some 'conventional' approaches might underestimate often high return levels of rainfall. The data analyzed are from Portugal.

Keywords: rainfall; extremes; multifractals.

1. Introduction

Rainfall is extremely non-linear and variable in time and space. No satisfactory detailed modeling of the complexity of this process has yet been achieved. The "conventional" models frequently used for studying extreme rainfall events are models developed within non-scaling frameworks; they study one scale independent of the other. Thus, to model rainfall two or more different distributions are often required to fitting different regimes such as the "low-intensity", the "regular" and the "extreme" events. Usually these models involve only weak variability; they assume often, for example, that the behavior of the probability tails is exponential. The assumption of weak variability is likely not able to characterize the strong fluctuations in rainfall. All of these are factors that limit the full statistical characterization of rainfall.

In this work an alternative approach to studying rainfall is used. This approach is developed within a scaling framework and assumes, therefore, invariance of properties across scales. It uses multifractal theory and models to characterizing rainfall.

2. The rainfall data

This work focuses on the analysis of 23 years of hourly rainfall from Lisbon (Geofísico), Portugal, covering the period 1980-2002. The geographic coordinates of the station are latitude 38° 43' N and longitude 09° 09' W; altitude is 77 m. The study area lies in the transitional region between the sub-tropical anticyclone and the sub-polar depression zones. The rainfall climate is characterized by large inter-annual variability and exhibits strong seasonal variability. During the period covered by the data the average annual rainfall was 714 mm, the maximum hourly precipitation was 43 mm, and the maximum daily rainfall recorded was 94 mm.

3. Brief introduction to multifractal theory and tools

Multifractal theory has the potential to assess the full range of fluctuations in rainfall and it offers a single framework to deal with the different regimes (e.g. Schertzer and Lovejoy, 1989, 1991; Lovejoy and Schertzer, 1991). The theory is based on the invariance of properties across scales, and it takes into account the persistence of the variability of the process over a range of scales; it is developed in a non-dimensional framework.

In general terms, the scale-invariant behavior embedded in multifractals leads to a class of scaling rules (i.e. power laws) characterized by scaling exponents. The multifractal temporal structure of rainfall can be investigated by studying the (multiple) scaling of the statistical moments of the rain rates (Schertzer and Lovejoy, 1987). The scaling behavior is described by the moments scaling exponent function $K(q)$ that satisfies:

$$\langle R_\lambda{}^q \rangle \approx \lambda^{K(q)} \tag{1}$$

where λ is related to the resolution and $<R_\lambda{}^q>$ is the (ensemble) average q^{th} moment of the flow rate on a scale specified by λ. Parameter λ is called scale ratio and is the ratio between the largest scale of interest and the scale of homogeneity of the data or process. In Eq. (1) the notion of moment can be generalized to any real value q. The scaling of the moments can be tested with log-log plots of the average q^{th} moment of the rain rate R_λ, observed on scales of different levels of resolution λ, against the scale ratio λ. The empirical scaling functions $K(q)$, in Eq. (1), are obtained from the regression lines of log($<R_\lambda{}^q>$) against log(λ) for various moments q of the rain rates.

A Legendre transform (Frisch and Parisi, 1985) establishes a one-to-one relation between orders of singularities γ of the intensities of a process and statistical moments q. This implies that a given rainfall singularity, described by $R_\lambda = \lambda^\gamma$, gives a dominant contribution to the q^{th} order moment ($q = c'(\gamma)$). The function $c(\gamma)$ is the multifractal scaling exponent function of the probability distributions of the process intensity and is called codimension function. Figure 1 illustrates schematically the relation between the orders of singularity γ of the intensities R_λ of a multifractal process in the embedding space *X* and the scale ratio λ; singularities γ_1 and γ_2 express two levels of intensity of the process.

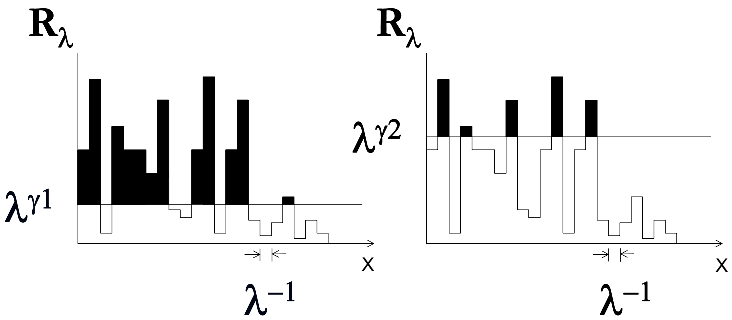

Fig. 1. Schematic illustration of the relation between the orders of singularity γ of the intensities R_λ of a multifractal process in the embedding space *X* and the scale ratio λ, defined as the quotient between the largest scale of interest and the homogeneity scale. The singularities γ_1 and γ_2 express two levels of intensity of the process.

The multifractal singular behavior at the small-scale limit (i.e., $R_\lambda \to \infty$ as $\lambda \to \infty$) leads to divergence of moments: $<R_\lambda{}^q> = \lambda^{K(q)} \to \infty$, for all moments $q > q_D$ (it is $K(q) > 0$ for $q > 1$). Please, note that there is equivalence between the divergence of moments (for $q > q_D$) and the algebraic fall-off of the probability distribution for extreme events (e.g. Feller, 1971; Mandelbrot, 1974; Schertzer and Lovejoy, 1985; Lovejoy and Schertzer, 1985). The tail of the probability law determines the relative frequency of extreme behavior; the slope of this tail is the critical order for divergence of statistical moments, q_D:

$$\Pr(R_\lambda > s) \approx s^{-q_D} \tag{2}$$

where s is a sufficiently large intensity-threshold. The smaller the exponent q_D, the more extreme is the fluctuation of the process. For rainfall, multifractal theory implicitly assumes the presence of heavy tails (i.e., power-law tails) in the probability distribution of the intensities. This approach is innovative and may have important implications in engineering design.

4. Analysis and results

The multifractality of rainfall is investigated here by testing the scaling behavior of the statistical moments of the rain rates. This scaling behavior can be tested with log-log plots of the average q^{th} moment of the rain intensity R_λ, observed on scales of different levels of resolution λ, against the scale ratio λ. For the data from Lisbon, Fig. 2 (left) shows these log-log plots for time scales from 1 hour (λ=4096) up to approximately 5.7 months (λ=1). The plots are for moments q larger than 1 and smaller than 1.

The power-law behavior observed in the moments´ plots confirms the presence of scale invariance in the temporal structure of rainfall over a large range of scales. The scaling range seems to extend from one hour up to about 11 days. Other studies of higher resolution data have shown that the lower limit of the scaling range is of the order of minutes (e.g. Olsson, 1995; de Lima, 1998; de Lima and Grasman, 1999). The upper limit of this scaling regime seems to be affected by local synoptic regimes.

The multifractality of rain is revealed by the different scaling exponents associated with the various moments q. The empirical scaling function $K(q)$, in Eq. (1), which describe the statistics of precipitation, are obtained from the regression lines fitted to the moments' plots in Fig. 2 (left), over the relevant range of moments q of precipitation intensity. This $K(q)$ function is shown in Fig. 2 (right); the function has both non-linear and linear sections. The non-linear section is a concave increasing function. Turning our attention to the large rain rate part of the statistics, we see that $K(q)$ is linear for q exceeding a critical value q_{crit}. Such a discontinuity in the first or second derivative of $K(q)$ arises either because of divergence of moments at q_{crit} (called q_D in this first order case) or simply due to the inadequate sample size (at q_s, in the second order case), so that all moments larger than q_s are determined by the largest value present in the sample (see e.g., Schertzer and Lovejoy, 1991).

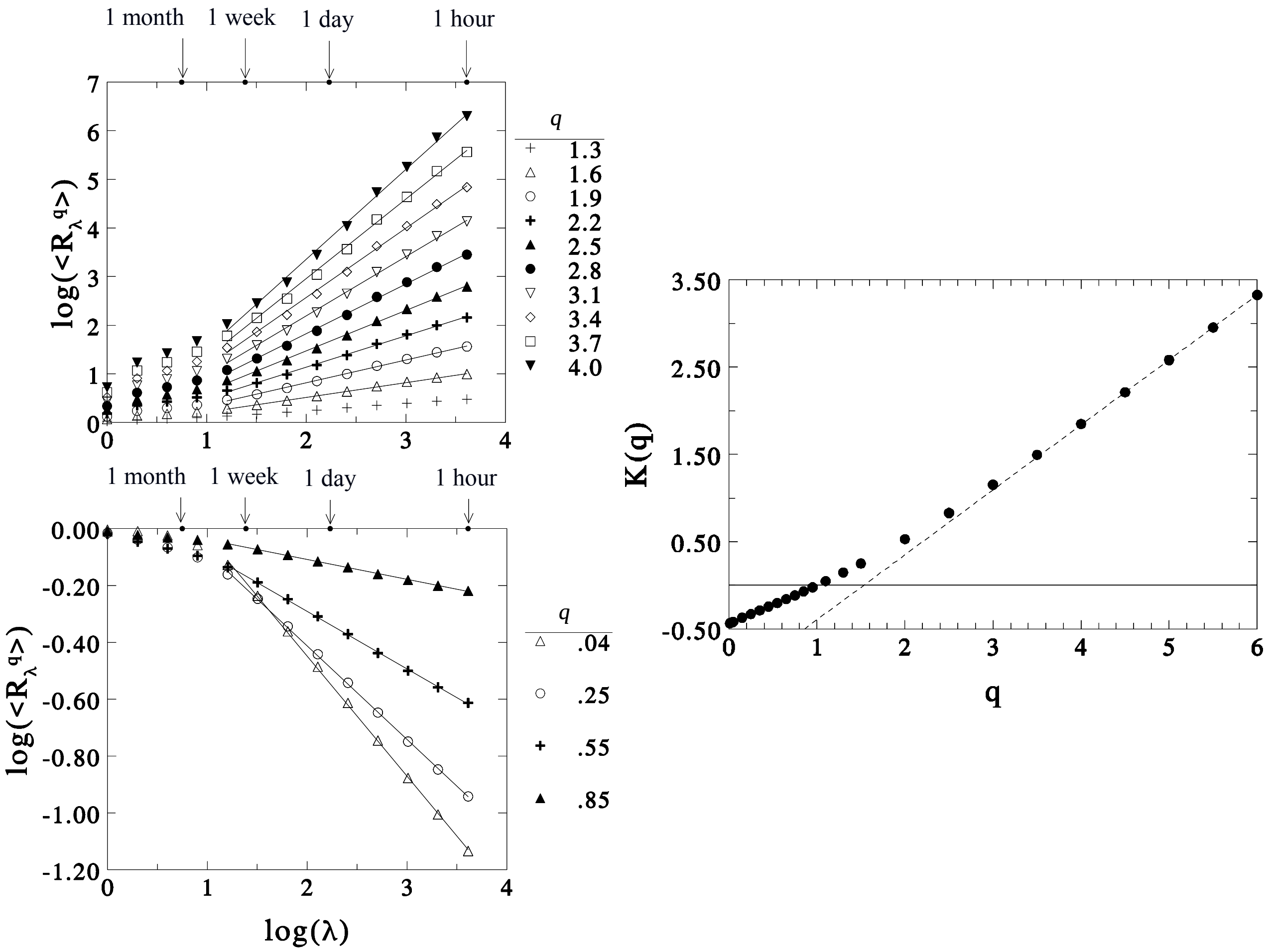

Fig. 2. Log-log plots of the average q^{th} moments of the precipitation intensity R_λ on scales between 1 hour (λ=4096) and 5.7 months (λ=1), against the scale ratio λ are shown on the left: (top) for moments larger than 1 and (bottom) for moments smaller than 1. The corresponding moments scaling function $K(q)$, for the range of scales from 1 hour up to approximately 11 days, is shown on the right hand side.

Thus, linear sections in both multifractal scaling exponent functions, $K(q)$ and $c(\gamma)$, correspond to a special type of statistical behavior that can be explained by divergence of moments above a certain critical order (i.e., for all moments $q > q_D$). A true divergence of moments would only be observed for infinite sample sizes; empirically, we have finite samples (and finite empirical moments) and this phenomenon is manifested by discontinuities in the empirical multifractal exponent functions. The divergence of moments means in this case that the empirical moments increase without limit as the sample size increases. This statistical behavior occurs because the sum of independent contributions is determined by the largest of the contributions (i.e., rare events will have dominant contributions).

We can thus determine the corresponding largest singularity present in the finite rainfall sample from $\gamma_{max} = max(K'(q))$. The slope of the linear section of the moments scaling function, for moments $q > q_D$, give an estimate of the largest order of rainfall singularity γ_{max} : 0.74. The study of the empirical function $K(q)$ near q = 0 yields estimates of the codimension $c(\gamma_{min}) = -K(0)$ of 0.43. This value is related to a minimum non-zero observation of the precipitation process, of singularity γ_{min}. The function $c(\gamma)$ at γ_{min}, $c(\gamma_{min})$, is the codimension of the "support" of the precipitation process on the temporal axis.

Estimates of the critical moment q_D can be obtained from the analysis of the precipitation intensity probability distribution functions. Figure 3 shows examples of probability distribution functions of rain on different scales (1, 2, 4 and 8 hours). These functions exhibit algebraic tails (heavy tails), which indicates divergence of statistical moments for moments $q > q_D$. The critical order moment q_D, in Eq. (2), is given by the absolute value of the slope of the algebraic tail. The mean absolute value of the slope of the regression lines fitted to the tails of the empirical probability distributions is 3.3. This estimate of the critical moment q_D agrees with the behavior of the empirical functions $K(q)$ (see Fig. 2): the discontinuity in this function is observed at roughly the same critical moment.

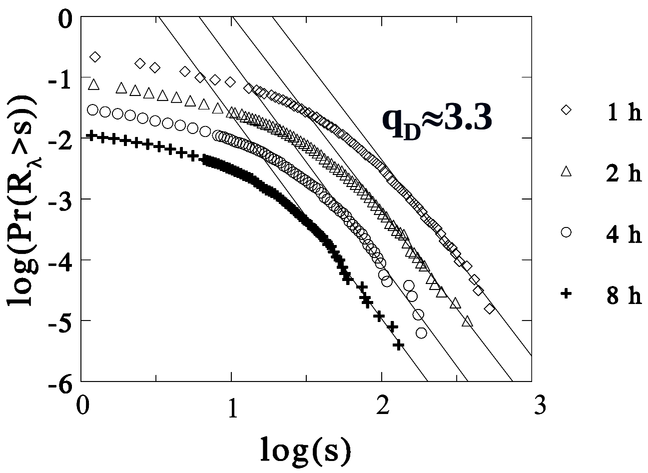

Fig. 3. Empirical probability distributions of precipitation on time scales of 1, 2, 4 and 8 hours, for the data from Lisbon, Portugal, in the period 1980-2002. The functions are displaced vertically, for clarity.

A similar behavior of the probability distributions of rain, described by the "heavy" tails, was also observed, among others, by Ladoy *et al*. (1991), Hubert *et al*. (1993), Olsson (1995), Tessier *et al*. (1996), de Lima (1998), de Lima *et al*. (2002), Douglas and Barros (2003), de Lima and de Lima (2009). These authors used rain data with different backgrounds, climate, record period and temporal resolutions. Most estimates of the critical moment found in the literature are about 3, although there are different reports.

In general, this statistical behavior may indicate that, in some cases, the probability of exceeding certain high-intensity events is greater than the probability predicted by more 'conventional' models characterized by exponential tails (e.g. Gumbel distribution): power-law tails fall-off slower that exponential tails.

5. Concluding remarks

Results illustrate that the scaling framework used in this work provides tools that are able to handle large rainfall dynamic ranges and quantify the variability in rainfall, taking into account the behavior observed across a large range of scales. Thus multifractal approaches bring together the properties of rainfall observed at different scales and overcome the problem of using different models to describe data of different resolutions; multifractals offer a single framework to deal with different regimes such as the “low-intensity”, the “regular” and the “extreme” events.

Multifractals contain singularities of extreme orders which are related to algebraic decays of the extreme events (i.e. algebraic tails of the probability distributions); these are associated with processes’ “violent” character. This type of behavior was also revealed by the data analyzed in this work. Heavy tails indicate that the probability of exceeding certain critical levels and events is much greater than the probability predicted by more “conventional” models (see e.g. Ladoy *et al*., 1991). These facts should be taken into account in engineering studies and design procedures.

Acknowledgements

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